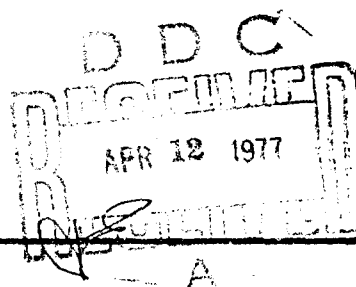WH0I-77-12

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HEAVE AND ROLL RESPONSE OF FREE FLOATING BODIES
OF CYLINDRICAL SHAPE

By

H. O. Berteaux
R. A. Goldsmith
W. E. Schott, III

February 1977

TECHNICAL REPORT

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The basic concepts of harmonic analysis and statistics used in the method are first briefly reviewed. The report then presents a detailed derivation of the linear heave and roll response amplitude operators (Cont.)

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The second part of the report reviews the computation procedure and the program's logic. It gives a detailed set of instructions for the program users, reviews the program's capabilities and limitations, and presents three case studies.

The heave and roll response programs are written for use with XEROX SIGMA 7 computers. Program listings are given in the appendix.

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TECHNICAL REPORT

Approved for Distribution *Melvin A. Rosenfeld*

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Department of Ocean Engineering

ABSTRACT

The following report describes a computer solution to help predict the heave and roll response of free floating bodies of cylindrical shape when excited by random seas with known spectra.

The basic concepts of harmonic analysis and statistics used in the method are first briefly reviewed. The report then presents a detailed derivation of the linear heave and roll response amplitude operators, that is the expressions of the vertical and angular displacements produced by a simple harmonic wave of one foot amplitude.

The second part of the report reviews the computation procedure and the program's logic. It gives a detailed set of instructions for the program users, reviews the program's capabilities and limitations, and presents three case studies.

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Acknowledgments

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1.0 PROBLEM STATEMENT

The heave and roll motion of a cylindrical body of constant cross section when excited by a simple harmonic wave is a relatively straight-forward problem. However, very few buoys can be realistically modeled as a pillbox or a telephone pole. Most spar buoys are made of circular cylinders of varying diameters (see Fig. #1). Some spar buoys extend to considerable depths below the water level. Furthermore, most sea-ways are not made of regular harmonic waves of single frequency and amplitude and in general irregularity and randomness of the sea surface will prevail.

The objective of this report is to present a method which can be used to compute reasonable expectations of vertical and angular displace-ment that a complex shape buoy will experience when free floating in a random stationary seaway.

The computer solution presented in this report was originally derived to investigate the dynamic behavior of specific spar buoys used by the Woods Hole Oceanographic Institution. This solution is here presented in a generalized form, with the hope that it becomes a con-structive addition to the solutions already in the literature.

2.0 THEORETICAL BACKGROUND

2.1 Statistical Response of Floating Bodies to Ocean Waves Excitation

Readers unfamiliar with the probabilistic theory of ship and buoy dynamics should resort to References I, II, and III for a theoretical introduction to the subject.
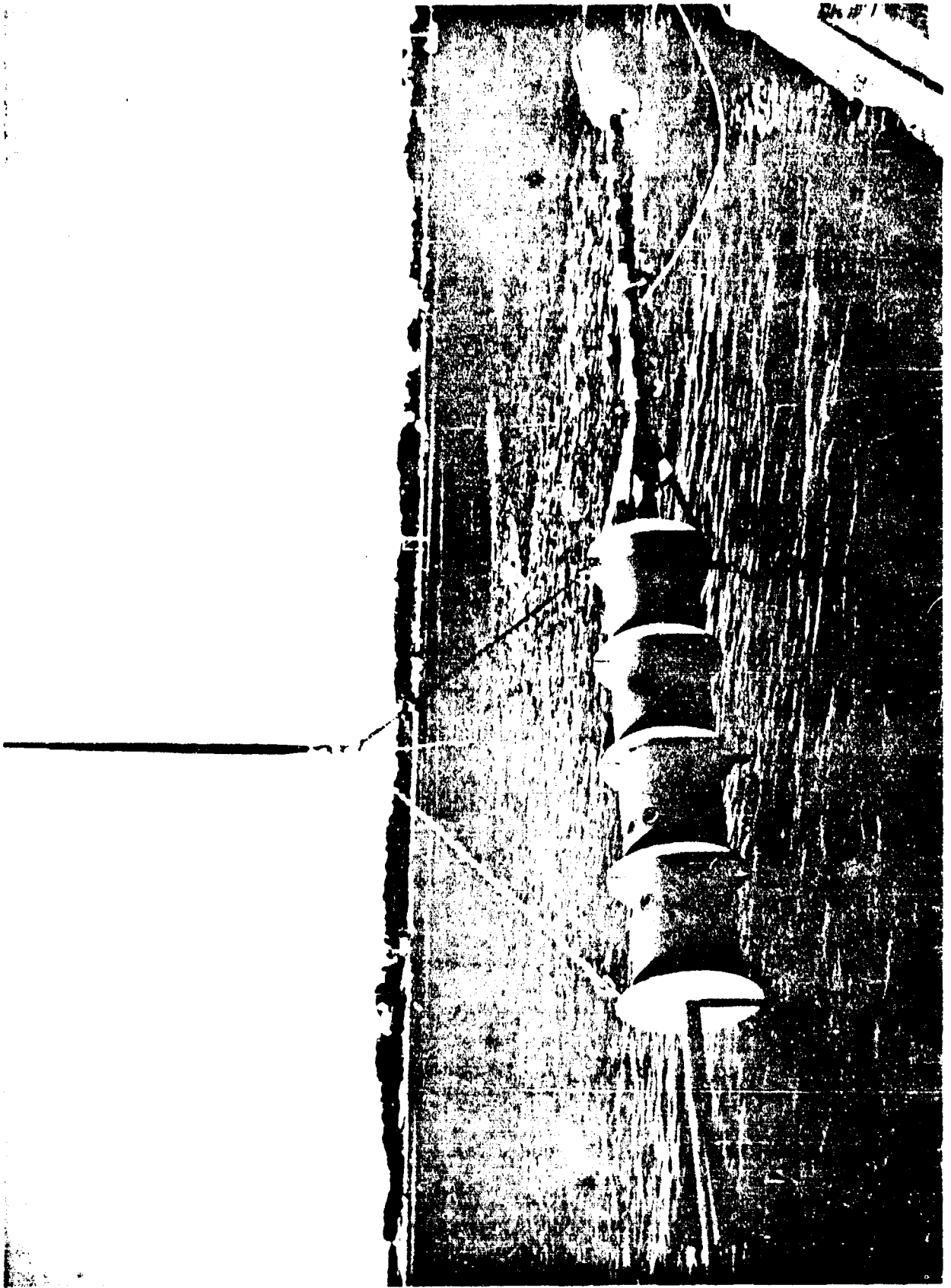
Fig. No. 1

Basic concepts borrowed from this theory and used in the formulation of the heave and roll computer programs described in this report are hereafter summarized.

If the probability density function $p(x)$ of the wave amplitudes "x" for a given seaway can be explicitly expressed, then the expectation of certain values of wave amplitudes can be directly computed.

For example:

- The most probable amplitude x_m is the value of x for which

$$\frac{d}{dx} p(x) = 0 \qquad (2.1.1)$$

- The average amplitude $\bar{x}$ is given by

$$\bar{x} = \int_0^\infty x\, p(x)\, dx \qquad (2.1.2)$$

- The average of a fraction $f\left(0 \leq f \leq 1\right)$ of wave amplitudes larger than a given amplitude x_0 can be obtained

$$\bar{x}_f = \frac{\int_{x_0}^\infty x\, p(x)\, dx}{\int_{x_0}^\infty p(x)\, dx} = \frac{1}{f} \int_{x_0}^\infty x\, p(x)\, dx \qquad (2.1.3)$$

etc.

When certain restrictive conditions prevail, wave amplitudes have been found to follow a Rayleigh distribution given by:

$$p(x) = \frac{2x}{\overline{x^2}}\, e^{-\frac{x^2}{\overline{x^2}}} \qquad (2.1.4)$$

where $\overline{x^2}$ is the mean square value of the wave amplitudes.

This probability density function has been used to compute the expectation of particular wave amplitude means and maxima. Results of these computations are found to be proportional to the root mean square $\sqrt{\overline{x^2}}$ of the wave amplitudes. Those retained in this study are summarized in Tables 1 and 2, "Value of expected means" and "Value of expected maxima."

Now let $\mathcal{S}_{(\omega)}$ be the spectral density function of the wave amplitudes of the given sea way. Assuming the seaway to be stationary, then the integral over all positive frequency ranges of $\mathcal{S}(\omega)$ is equal to the mean square value of the wave amplitudes, i.e.

$$\overline{x^2} = \int_0^\infty \mathcal{S}(\omega)\, d\omega \qquad (2.1.5)$$

This result can be used to compute $\overline{\widetilde{x^2}}$ and $\sqrt{\overline{x^2}}$. The value of $\sqrt{\overline{x^2}}$ thus obtained can in turn be used to compute the expected wave amplitude means and maxima listed in Tables 1 and 2.

From the definition of the mean square value

$$\overline{x^2} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{i=N} x_i^2$$

and the result (2.1.5) it is clear that the quantity

$$\lim_{d\omega \to 0} \sqrt{\overline{\mathcal{S}(\omega)\, d\omega}} \qquad (2.1.6)$$

is proportional to the amplitude of the elementary component wave of the spectrum with frequency ω_n.

Now if $\mathcal{Y}(\omega)$ is the expression of the linear response of a

Table 1

Wave Amplitude Means

Fraction, f, of Largest Amplitudes Considered	Mean Values $\overline{\eta_f} \div \sqrt{\overline{\eta^2}}$
0.01	2.359
0.10	1.800
0.333	1.416
0.50	1.256
1.00	0.886

Table 2

Expected Maximum Amplitudes

Number of Waves	Maximum Wave Amplitudes $\eta_{max} \div \sqrt{\overline{\eta^2}}$
50	2.12
100	2.28
500	2.61
1,000	2.78
10,000	3.13
100,000	3.47

free floating body to a simple harmonic wave of unit amplitude

and frequency ω , then clearly the quantity

$$\lim_{dw \to 0} Y(\omega) \sqrt{S(\omega_m) \, d\omega}$$

(2.1.7)

is proportional to the amplitude of the body response to the

elementary component wave of frequency ω_m .

It thus follows that

$$Y^2_{(\omega)} \, S(\omega_m) \, d\omega$$

(2.1.8)

is proportional to the amount of the response mean square

value contained in the frequency band $d\omega$ centered at ω_m .

The response mean square value $\overline{r^2}$ is therefore given by:

$$\overline{r^2} = \int_0^\infty Y^2_{(\omega)} \, S_{(\omega)} \, d\omega$$

(2.1.9)

The response $Y_{(\omega)}$ of the body being linear, the probability

density function of the response will also follow the probability

density function of the wave amplitudes. Thus the results tabu-

lated in Tables 1 and 2 can be used again, together with expression

(2.1.9) to compute statistical means and maxima of body response

amplitude.

For example, the average of the one third highest response

amplitudes will be given by

$$\overline{r_{\frac{1}{3}}} = 1.416 \sqrt{\overline{r^2}}$$

with
$$\sqrt{\overline{r^2}} = \sqrt{\int_0^\infty Y^2_{(\omega)} \, S_{(\omega)} \, d\omega}$$

(2.1.10)

Empirical formulation of wave amplitude spectra used in the computer program are:

$$S(\omega) = \frac{16.875}{\omega^5} e^{\frac{-9.7 \times 10^4}{V^4 \omega^4}} \qquad \text{ft}^2 - \text{sec (Pierson Moskowitz)} \quad 2.1.11$$

where V is the wind speed (knots).

$$S(\omega) = \frac{525 H_s^2}{T_s^4 \omega^5} e^{\frac{-1050}{T_s^4 \omega^4}} \qquad \text{ft}^2 - \text{sec (Bretschneider)} \qquad 2.1.12$$

$$S(\omega) = \frac{345 H_s^2}{T_s^4 \omega^5} e^{\frac{-630}{T_s^4 \omega^4}} \qquad \text{ft}^2 - \text{sec (I.S.S.C.)} \qquad 2.1.13$$

In formula (2.1.12) and (2.1.13) H_s is the significant wave height (feet) and T_s is the significant wave period (seconds).

2.2 <u>Derivation of the Heave and Roll Response Amplitude Operators (RAO)</u>

2.2.1 <u>Heave response</u>

2.2.1.1 <u>Initial conditions</u>
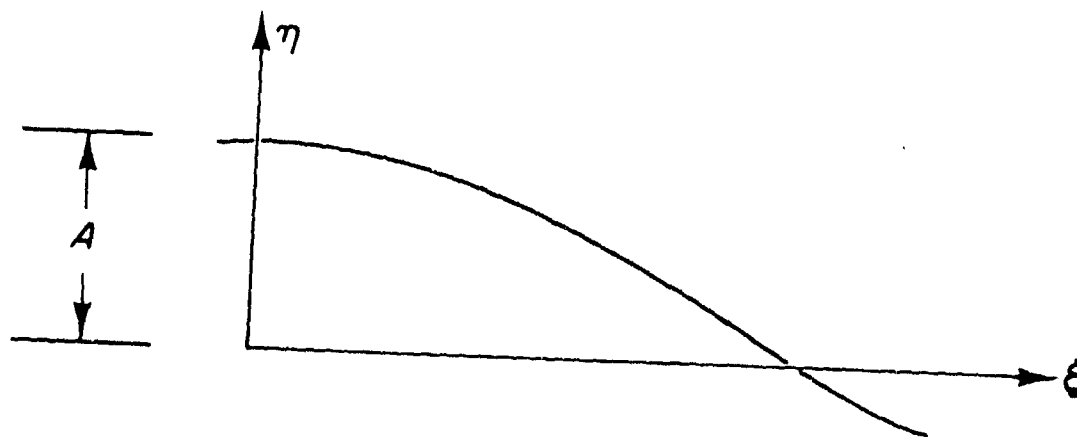
Fig. No. 2

Let us consider a simple harmonic, deep sea wave, as shown on Fig. No. 2. The coordinates of a point on the surface of this wave are given by:

$$\xi = A \sin (\omega t - K \zeta) \qquad (2.2.1)$$

$$\eta = A \cos (\omega t - K \zeta) \qquad (2.2.2)$$

If we select to observe this wave at $\zeta = 0$, then the parametric equation of the water particle motion around this point become:

$$\xi = A \sin \omega t$$

$$\eta = A \cos \omega t$$

where A is the wave amplitude, and ω the wave angular frequency.

We also know from the simple harmonic wave theory (Ref. I, pp. 14-27) that the parametric equations of water particles at any depth z below the mean water level would then be

$$\xi = A e^{-Kz} \sin \omega t$$

$$\eta = A e^{-Kz} \cos \omega t$$

where K is the wave number. $K = \dfrac{\omega^2}{g}$ for deep water waves, g being the gravity acceleration.

The vertical components of water particle velocity and acceleration would in turn be given by:

$$\dot{\eta} = -\omega A e^{-Kz} \sin \omega t$$

$$\ddot{\eta} = -\omega^2 A e^{-Kz} \cos \omega t$$

In this case, at time $t = 0 + \varepsilon$, the amplitude of the vertical

displacement of the water particles start to decrease from their
maximum value, the water particles vertical velocity component
starts to increase and is in the downwards direction, and the
water particles vertical acceleration component starts decreas-
ing and is also in the downwards direction.

As shown on Fig. No. 3, let χ be the distance from the
still water surface to the buoy water line. At time $t = 0 + \varepsilon$ the
buoy is assumed to move downwards, that is the distance χ is
increasing.

2.2.1.2 General equation of heave motion

The equation of heave motion will be obtained from:

$$\sum_i F_i = (m + m')\ddot{x} = m_v \ddot{x} \qquad (2.2.3)$$

where $\sum_i F_i$ = sum of the vertical forces applied to
the buoy,

m = mass of the buoy

m' = added mass of the buoy due to the water entrained
in the vertical direction

$m_v = m + m'$ = virtual mass of the buoy (in the vertical
direction).

2.2.1.3 Expression of the forces applied to the buoy

The vertical forces applied to the buoy are:

- Its weight "W"

- The resultant "$\underline{P}$" of the pressure forces exerted
  by water particles on the top and the bottom plates of
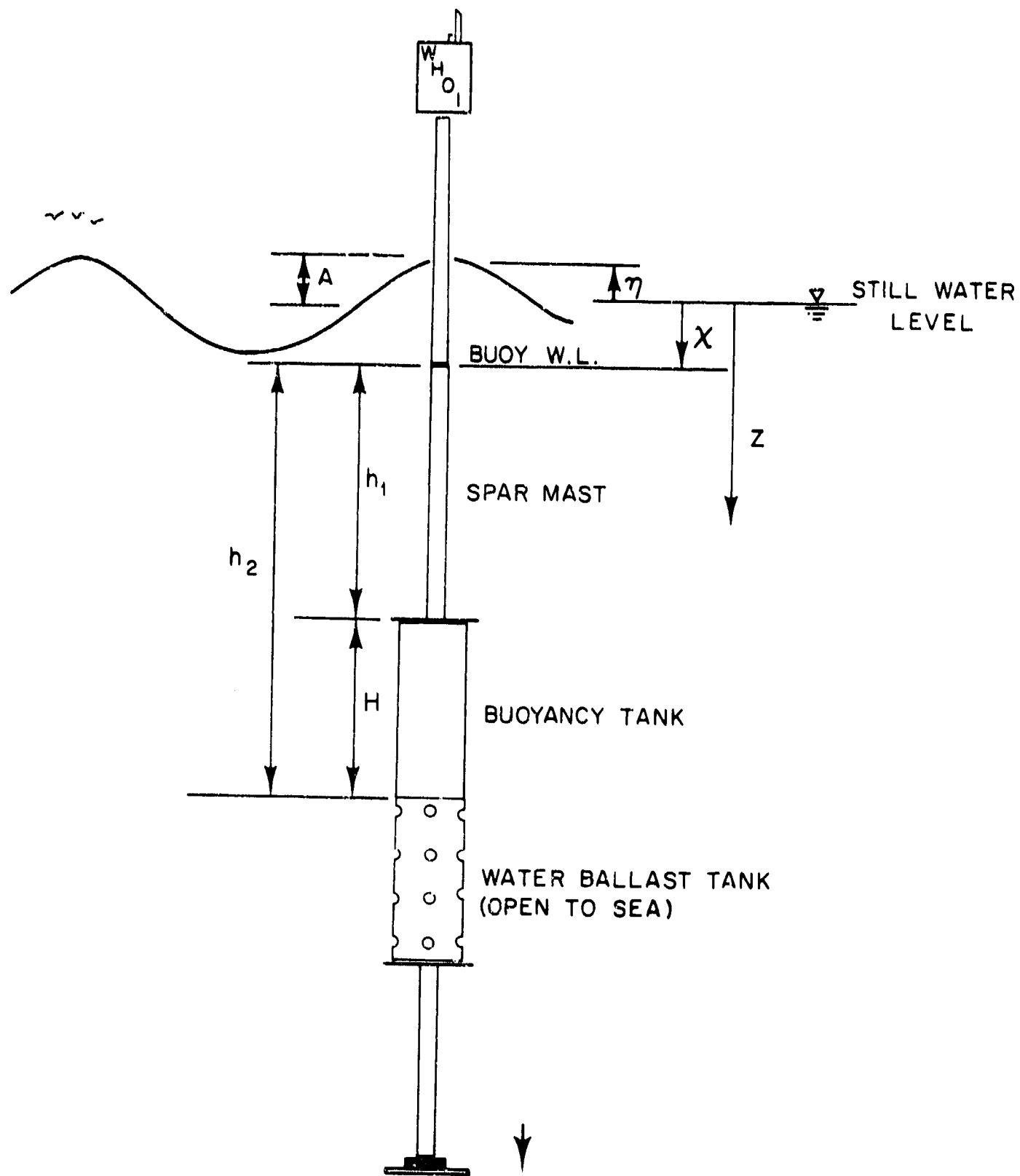
Fig. No. 3

the watertight compartments of the buoy

- The damping force "$\coprod$" resulting from the water

   opposing the buoy vertical motion

- The friction force "G" exerted by the water particles

   vertical velocity on the buoy

- The inertial force "I" exerted by the water particles

   vertical acceleration on the buoy.

Forces in the direction of increasing χ (downwards) will be considered positive.

The expression of these forces is obtained as follows:

- <u>Pressure force "P".</u>

With the initial conditions assumed, the pressure p at a depth

Z is given by:

$$p = \rho g \left(z + A e^{-\kappa z} \cos \omega t \right)$$

To help find a general expression for the resultant P , let us

consider the spar buoy shown in Fig. No. 3.

At the bottom of the buoyancy tank

$$Z = \chi + h_2$$

where h_2 = depth of the bottom plate below buoy water line

Assuming $\chi \ll h_2$, the upwards pressure force P_B on the tank

bottom is thus given by

$$P_B = \rho g \left\{ \chi + h_2 + A e^{-\kappa h_2} \cos \omega t \right\} S_B$$

where S_B = area of bottom plate subjected to water pressure

(the entire area of the plate in this case).

Similarly the pressure force P_T on the top plate of the buoyancy

tank is given by

$$P_T = \rho g \left\{ x + h_1 + A e^{-kh_1} \cos \omega t \right\} S_T$$

where S_T = area of top plate subjected to water pressure.

If the spar mast has a cross section S_M and is watertight then obviously $S_T = S_B - S_M$

The resultant $\underline{P}$ will be the difference between the bottom pressure force and the top pressure force. Being in the upwards direction, $\underline{P} = -(P_B - P_T)$ i.e.

$$\underline{P} = -\rho g \left\{ (S_B - S_T) x + (h_2 S_B - h_1 S_T) - A \cos \omega t (S_B e^{-kh_2} - S_T e^{-kh_1}) \right\}$$

The constant terms in the expression of the pressure force must equal the buoy static weight $\overline{W}$. This can be easily established.

Noting that

$$h_2 = h_1 + H$$

where H = length of the buoyancy tank, the constant terms

$$\rho g \left(S_B h_2 - S_T h_1 \right)$$

can be written:

$$\rho g \left(\bar{S}_B (h_1 + H) - S_T h_1 \right] = \rho g \left[(\bar{S}_B - S_T) h_1 + S_B H \right]$$

which obviously is the sum of the weight of the water displaced by the immersed portion of the mast and by the buoyancy tank under equilibrium conditions, and therefore is equal to the buoy weight.

The sum of the weight force and the pressure force can then be in general expressed by:

$$\underline{P} + W = -\rho g \left(S_B - S_T \right) x + \left(\rho g \sum_i' e^{-kh_i} S_i \right) A \cos \omega t \qquad (2.2.4)$$

where S_i is the surface at a depth h_i subjected to the

pressure. S_i is positive if the pressure exerted upon it
is in the upwards direction, and vice versa S_i is nega-
tive if the pressure exerted upon it is in the downwards
direction.

This expression can further be simplified and written:

$$P + W = -Cx + MA \cos \omega t$$

(2.2.5)

where

$$C = \rho g \left(S_B - S_T \right) = \rho g \, S_c$$

(2.2.6)

is the heave restoring force constant and

$$M = \rho g \sum_i S_i \, e^{-Kh_i} = \rho g \sum_i' S_i \, e^{-\frac{\omega^2}{g} h_i}$$

(2.2.7)

- Damping force "D".

The damping force D_i exerted by the water on a buoy component
"i" will be assumed to be directly proportional to the buoy
speed $\dot{x}$. It will therefore be of the form

$$D_i = -b_i \, \dot{x}$$

(2.2.8)

where b_i is the linearized coefficient of damping

associated with buoy heave motion.

It can be shown (see Appendix I) that the general expression of

linearized damping coefficients "d" for periodic motion of

amplitude X and frequency ω is of the form

$$d = \frac{4}{3\pi} \, \rho \, C_D \, S X \omega$$

(2.2.9)

where ρ = water mass density = 2 slugs/ft^3

C_D = conventional drag coefficient

S = area normal to the flow.

In order to keep the differential equation of heave motion linear, an arbitrary constant value $\overline{X}_b$ of average heave motion must be selected to compute the linearized damping coefficients b_i. The value of $\overline{X}_b$ selected is left as an input for the program users. One can use, for example, a reasonable fraction of the average wave amplitude for the sea state considered in a given study.

The expression of b_i then becomes

$$b_i = \frac{4}{3\pi}\rho\, C_{D_i} S_i\, \overline{X}_b\, \omega = \omega\, b_i' \qquad (2.2.10)$$

The total damping force is thus finally

$$D = \sum_i{}' D_i = -\omega \dot{x} \sum_i \frac{4}{3\pi}\rho\, C_{D_i} S_i\, \overline{X}_b$$

or simply

$$D = -B\dot{x}$$

with

$$B = \omega \sum_i{}' \frac{4}{3\pi} C_{D_i} S_i\, \overline{X}_b = \omega \sum_i{}' b_i' \qquad (2.2.11)$$

- <u>Wave induced drag force "G".</u>

The drag force G_i resulting from the water particle impinging with a velocity $\dot{\eta}$ on a buoy component "i" is also assumed to be linearly proportional to $\dot{\eta}$. It therefore will be expressed by

$$G_i = c_i \dot{\eta} \qquad (2.2.12)$$

where c_i is the linearized coefficient of drag associated with water particle velocity.

Following previous reasoning the expression of c_i will be

- 12 -

given by

$$C_i = \frac{4}{3\pi} \rho \, C_{Di} \, S_i \, \overline{X_c} \, \omega = \omega \, C_i'$$ (2.2.13)

where $\overline{X_c}$ is now the arbitrary average value of wave

amplitudes retained for the particular study.

A comment should also now be made regarding the water particle

velocity $\dot{\eta}$. It will be recalled that the expression of $\dot{\eta}$ is

$$\dot{\eta} = - A\omega e^{-\kappa z} \sin \omega t$$

In the case of a plate or a cylinder of small height placed at a

distance h below the buoy $W.L.$ and if $x \ll h$, then $z \cong h$

and the speed of the water particles acting on this plate is well

established. On the other hand, if the cylinder is one of con-

siderable height, as for example the buoyancy tank shown in

Fig. No. 3, then the speeds at the top and at the bottom must be

somehow averaged and replaced by a unique equivalent speed.

For simplicity, one could consider this averaged speed to be the

speed at the depth of the cylinder midpoint. For the buoyancy

tank previously mentioned this speed would then be:

$$- A\omega e^{-\kappa(h_1 + \frac{H}{2})} \sin \omega t$$

More appropriate values of equivalent depths could also be devised.

With these remarks in mind, the expression of the friction force

becomes

$$G = \sum_i' G_i = - A\omega \sin \omega t \sum_i' \frac{4}{3\pi} \rho \, C_{Di} \, S_i \, \overline{X_c} \, \omega \, e^{-\kappa z_i}$$

z_i being the true or the equivalent depth of the

component $"i"$.

More simply written,

$$G = -NA\omega \sin \omega t \qquad (2.2.14)$$

with

$$N = \omega \sum_i \frac{4}{3\pi} \rho \, \mathcal{G}_i \, \mathcal{S}_i \, \bar{X}_c \, e^{-kz_i} = \omega \sum_i c_i' \, e^{-\frac{\omega^2}{g}z_i} \qquad (2.2.15)$$

- Inertial force "I".

The inertial force I_i produced by the water particle accelera-tion $\ddot{z}$ on a component "i" of the buoy is given by

$$I_i = m_i' \, \ddot{z}$$

where m_i' is the added mass of the component "i" and is given by

$$m_i' = \rho \, C_{m_i} V_i$$

with C_{m_i} = added mass coefficient of component "i"

V_i = volume of the $i\text{th}$ component (ft^3).

The values of C_{m_i} and V_i depending of course on the dimen-sions and shape of the component "i", are left as an input for the program users.

The remarks on the averaged value of the water particle speed also apply for the water particle acceleration.

The expression of the inertial force "I" is therefore given by

$$I = \sum_i' I_i = -A\omega^2 \cos \omega t \sum_i' m_i' \, e^{-kz_i} \qquad (2.2.16)$$

or simply,

$$I = -QA\omega^2 \cos \omega t$$

with

$$Q = \sum_i' m_i' \, e^{-kz_i} = \rho \sum_i' C_{m_i} V_i \, e^{-\frac{\omega^2}{g}z_i} \qquad (2.2.17)$$

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2.2.1.4 Expression of the differential equation of heave

Using $\sum_i F_i = m_v \ddot{x}$ yields:

$$-C_x + MA\cos\omega t - B\dot{x} - NA\omega\sin\omega t - QA\omega^2\cos\omega t = m_v \ddot{x}$$

or,

$$C_x + B\dot{x} + m_v \ddot{x} = A\left\{(M - \omega^2 Q)\cos\omega t - N\omega\sin\omega t\right\}$$

This expression can be further reduced to:

$$C_x + B\dot{x} + m_v \ddot{x} = \overline{F}_o \cos(\omega t + \sigma) \tag{2.2.18}$$

where $\overline{F}_o$, the exciting force is given by

$$F_o = A\sqrt{(M - \varphi\omega^2)^2 + (N\omega)^2} \tag{2.2.19}$$

and σ , the phase angle between the wave and the force is

given by

$$\sigma = \tan^{-1}\frac{N\omega}{M - \omega^2 Q} \tag{2.2.20}$$

2.2.1.5 Expression of the heave response amplitude operator

Let us assume that a particular solution of the heave

equation is given by:

$$x = X_o \cos(\omega t + \psi)$$

where ψ is the phase angle between the exciting force and the

heave response. Then,

$$x = X_o(\cos\omega t \cos\psi - \sin\omega t \sin\psi)$$
$$\dot{x} = -X_o\omega(\sin\omega t \cos\psi + \cos\omega t \sin\psi)$$
$$\ddot{x} = -X_o\omega^2(\cos\omega t \cos\psi - \sin\omega t \sin\psi)$$

Introducing these values of x , $\dot{x}$, $\ddot{x}$ in the equation of

heave motion and ignoring for the moment the phase angle σ

yield:

$$CX_0 \left(\cos \omega t \cos \psi - \sin \omega t \sin \psi \right)$$
$$- BX_0 \omega \left(\sin \omega t \cos \psi + \cos \omega t \sin \psi \right)$$
$$- m_v X_0 \omega^2 \left(\cos \omega t \cos \psi - \sin \omega t \sin \psi \right) = F_0 \cos \omega t$$

Thus,

$$X_0 \left\{ (C - m_v \omega^2) \cos \psi - B \omega \sin \psi \right\} = F_0$$

and

$$X_0 \left\{ (-C + m_v \omega^2) \sin \psi - B \omega \cos \psi \right\} = 0$$

From the second result,

$$\tan \psi = \frac{-\omega B}{C - m_v \omega^2} \qquad (2.2.21)$$

Therefore

$$\sin \psi = \frac{-\omega B}{\sqrt{(C - m_v \omega^2)^2 + (\omega B)^2}}$$

and

$$\cos \psi = \frac{C - m_v \omega^2}{\sqrt{(C - m_v \omega^2)^2 + (\omega B)^2}}$$

Introducing these values of $\sin \psi$ and $\cos \psi$ in the

first result, yields:

$$X_0 = \frac{F_0}{\sqrt{(C - m_v \omega^2)^2 + (\omega B)^2}}$$

The expression of the heave response is thus finally given by

$$x = \frac{A\sqrt{(M-\omega^2 Q)^2 + (N\omega)^2}}{\sqrt{(C - m_v \omega^2)^2 + (\omega B)^2}} \cos(\omega t + \sigma + \psi) \tag{2.2.22}$$

The response amplitude operator being the ratio of the heave amplitude x by the wave amplitude A is thus in turn expressed by

$$R.A.O. = \frac{x}{A} =$$

$$\frac{\sqrt{\left(\rho g \sum_i S_i e^{-\frac{\omega^2}{g} h_i} - \omega^2 \rho \sum_i C_{m_i} V_i e^{-\frac{\omega^2}{g} z_i}\right)^2 + \left(\omega^2 \sum_i C_i' e^{-\frac{\omega^2}{g} z_i}\right)^2}}{\sqrt{(\rho g S_c - m_v \omega^2)^2 + \left(\omega^2 \sum_i b_i'\right)^2}} \tag{2.2.23}$$

2.2.1.6 Phase relationship between heave and wave

As previously established, the phase angle σ between wave and exciting force is given by

$$\sigma = \tan^{-1} \frac{\omega^2 \sum_i c_i' e^{-\frac{\omega^2}{g} z_i}}{\rho g \sum_i S_i e^{-\frac{\omega^2}{g} z_i} - \omega^2 \rho \sum_i C_{m_i} V_i e^{-\frac{\omega^2}{g} z_i}} \tag{2.2.24}$$

The phase angle ψ between the exciting force and the heave response is in turn given by

$$\psi = \tan^{-1} \frac{-\omega^2 \sum_i b_i'}{\rho g S_c - m_v \omega^2} \tag{2.2.25}$$

The phase angle ϕ between wave and heave response is finally given by $\phi = \sigma + \psi$ $\tag{2.2.26}$

2.2.2   Roll Response

2.2.2.1   Initial conditions

Let us consider again the simple harmonic, deep sea wave shown in Fig. No. 2.

The slope of this wave is given by:

$$\frac{\partial \eta}{\partial s} = KA \sin (\omega t - K s)$$

If we again select to observe this wave at $s = 0$, the expression of the slope $\underline{\phi}_s$ becomes

$$\underline{\phi}_s = KA \sin \omega t$$

The horizontal components of the velocity and acceleration of a water particle at a depth z will in turn be given by:

$$\dot{s} = \omega A e^{-Kz} \cos \omega t$$
$$\ddot{s} = -\omega^2 A e^{-Kz} \sin \omega t.$$

At the time $t = 0 + \varepsilon$ the magnitude of the horizontal particle velocity is maximum and is positive (i.e., in the direction of increasing ξ), the magnitude of the horizontal particle acceleration is minimum and in the opposite direction, and the magnitude of the slope is minimum and starts to increase.

Let $\ominus$ be the angle of roll, measured from the vertical in a clockwise direction. At time $t = 0 + \varepsilon$, the buoy will be assumed to roll in this direction, i.e., the angle of roll is increasing.

These initial conditions are depicted in Fig. No. 4. Rotation of the buoy is assumed to take place around the buoy center

VERTICAL

$\ominus$

HORIZONTAL

$\ddot{\xi}$

$\dot{\xi}$

ϕ

$\ominus - \phi$

$\oplus$

$t = 0 + \epsilon$
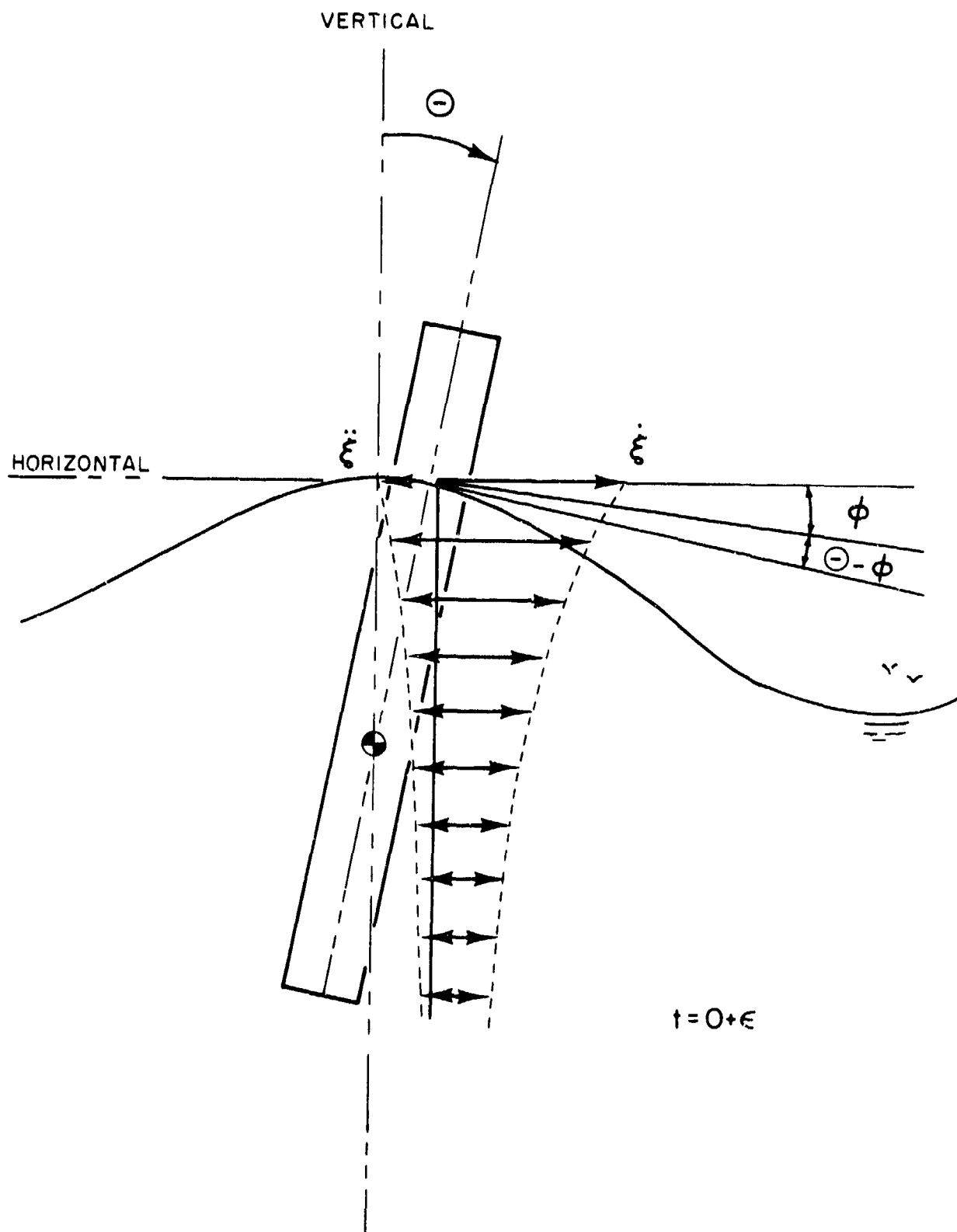
Fig. No. 4

of gravity.

2.2.2.2 General equation of roll motion

The equation of roll motion is given by:

$$\sum_i{}' \mathcal{M}_i = \left(I + I_F\right)\ddot{\theta} = I_v\,\ddot{\theta} \qquad (2.2.27)$$

where $\sum_i{}'\mathcal{M}_i$ = sum of the moments applied to the buoy

$\underline{I}$ = moment of inertial with respect to c.g. of buoy

I_F = added moment of inertia due to entrained water, also with respect to buoy c.g.

I_v = virtual moment of inertia = $I + I_F$

2.2.2.3 Expression of the moments applied to the buoy

Moments applied to the buoy are:

- righting moment caused by displacement of center of buoyancy, $\mathcal{M}_R$.

- damping moment due to buoy motion in the water $\mathcal{M}_D$.

- friction moment due to drag forces induced on the buoy by horizontal water particle velocity $\mathcal{M}_F$.

- inertia moment due to inertia forces induced on the buoy by horizontal water particle acceleration $\mathcal{M}_I$.

Clockwise capsizing moments will be considered positive, and vice versa counterclockwise righting moments will be considered negative.

The expression of these moments can be derived as follows:

- Righting moment - $\mathcal{M}_R$

The righting moment opposes buoy motion. Its value is

given by:

$$\mathcal{M}_R = -W g\overline{m}\left(\theta - \phi\right) = -W g\overline{m}\left(\theta - KA\sin\omega t\right) \quad (2.2.28)$$

where $\widehat{W}$ = buoy weight

$g\overline{m}$ = distance from buoy center of gravity to

buoy metacenter.

- Damping moment - $\mathcal{M}_D$

The drag forces due to buoy motion alone oppose the roll both

above and below the buoy center of gravity. Therefore, the

damping moment is negative. Its expression is derived as

follows:
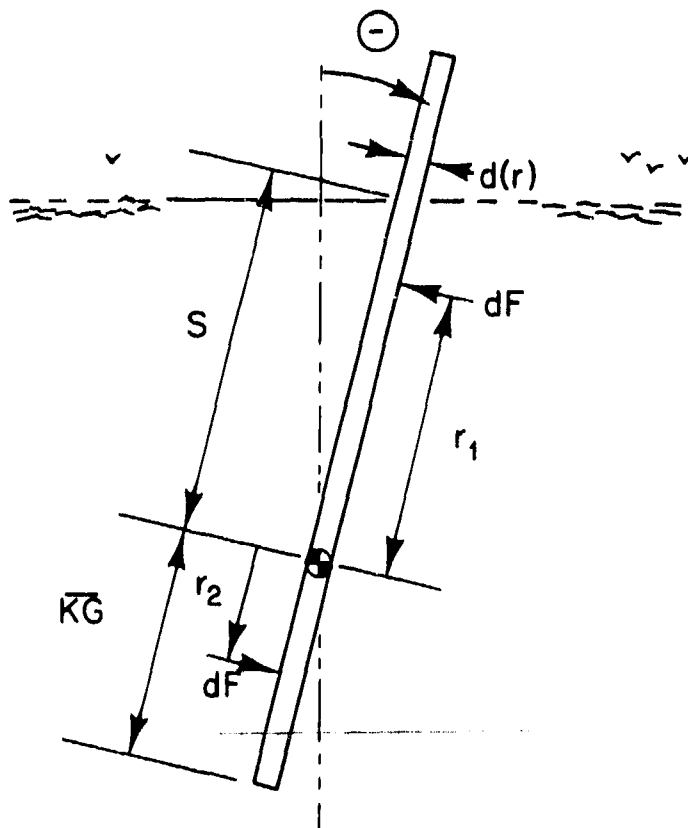
Fig. No. 5

Let us consider an elementary buoy section at a distance r from the buoy c.g. (See Fig. No. 5)

The elementary damping force on this element will be assumed to be of the form:

$$dF_D = b(r) \, r \, \dot{\theta} \cos \theta$$

or, for small angles of roll θ

$$dF_D = b(r) \, r \, \dot{\theta}$$

where $b(r)$ is a linearized damping coefficient again given by:

$$b(r) = \frac{4}{3\pi} \rho \, C_D \, S_{(r)} \, X_{(r)} \, \omega$$

with

ρ = fluid density = 2 slugs/ft^3

C_D = drag coefficient for cylinders, normal flow

$S_{(r)}$ = area across the flow = $d(r) \, dr$ with $d(r)$ the cylinder diameter at distance r

$X_{(r)}$ = amplitude of cyclic motion at distance r

$X_{(r)} = r\theta$ (in order to keep the equation of motion linear an arbitrary constant value of θ must be selected, say $\theta = \bar{\theta}$).

ω = angular frequency of cyclic motion, which under steady state conditions should equal the frequency of the exciting wave.

The expression of the damping force thus becomes:

$$dF_D = \left(\frac{4}{3\pi} \rho \, C_D \, \bar{\theta} \, \omega \, d(r) \, r \, dr \right) r \, \dot{\theta}$$

- 21 -

or

$$dF_D = \alpha \omega \, d(r) \, r^2 dr \, \dot{\theta}$$

where

$$\alpha = \frac{4}{3\pi} \rho \, C_D \, \bar{\theta} \qquad (2.2.29)$$

The moment of this elementary force is:

$$d\mathcal{M}_D = -r \, F_D = -\alpha \omega \, \dot{\theta} \, d(r) \, r^3 dr$$

and the total damping moment is found from

$$\mathcal{M}_D = -\dot{\theta}\alpha\omega \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1^3 dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2^3 dr_2 \right\}$$

or

$$\mathcal{M}_D = -B\dot{\theta} \qquad (2.2.30)$$

where

$$B = \alpha\omega \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1^3 dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2^3 dr_2 \right\} \qquad (2.2.31)$$

Appendix II outlines a method for computing these integrals.

- Wave drag moment - $\mathcal{M}_F$

Drag forces due to water particle velocity will tend to capsize
the buoy or to upright it depending on their point of application
with respect to the c.g.

The resulting moment will thus be positive above the c.g. and
negative below the c.g.
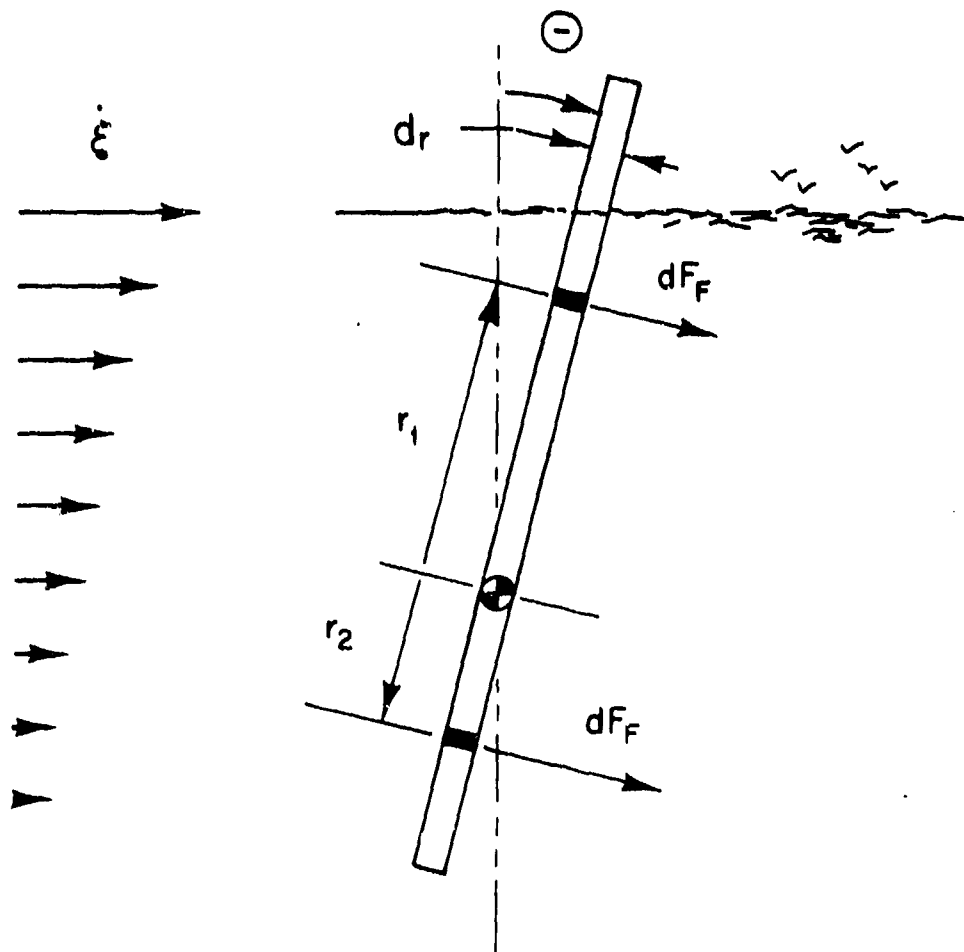
Fig. No. 6

Consider again an elementary buoy section of area $d(r) \, dr$
at a distance r from the buoy center of gravity (Fig. No. 6).
The elementary drag force due to the water horizontal velocity
on this elementary section will be assumed to be of the form:

$$dF_F = c(r) \, \dot{\xi}$$

where $c(r)$ the linearized damping coefficient, will be expressed
by:

$$c(r) = \frac{4}{3\pi} \rho \, C_D \, S(r) \, X(r) \, \omega$$

$X(r)$ in this case is the amplitude of the water particle cyclic

motion and is therefore given by

$$X(r) = A e^{-Kz}$$

Here again, in order to maintain linearity in the expression of the roll RAO, an arbitrary constant amplitude $\overline{A}_F$ must be selected. One could, for example, select the average amplitude $\overline{A}$ of the waves in the particular sea state.

With these remarks in mind, the expression of $C(r)$ can be written:

$$C(r) = \frac{4}{3\pi} \rho C_D \, d(r) \, dr \, \overline{A}_F \, \omega \, e^{-Kz}$$

or

$$C(r) = \omega \beta d(r) e^{-Kz} \, dr$$

where

$$\beta = \omega \frac{4}{3\pi} \rho C_D \overline{A}_F \qquad (2.2.32)$$

The expression of the elementary drag force dF_F thus becomes

$$dF_F = \beta A \omega \cos \omega t \, e^{-2Kz} \, d(r) \, dr$$

The moment of this elementary force is in turn:

$$dM_F = r \, dF_F = \beta A \omega \cos \omega t \, d(r) \, r e^{-2Kz} \, dr$$

Noting that the drag forces have a tendency to capsize the buoy when applied above the buoy c.g., and to upright the buoy when applied below the c.g., the expression of the wave drag moment becomes:

$$m_F = \beta A \cos \omega t \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1 e^{-2Kz} dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2 e^{-2Kz} dr_2 \right\}$$

or,

$$m_F = DA\omega \cos \omega t \qquad (2.2.33)$$

where

$$D = \beta \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1 e^{-2Kz} dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2 e^{-2Kz} dr_2 \right\} \qquad (2.2.34)$$

A method for the evaluation of the coefficient D is outlined

in Appendix III.

- Wave inertia moment - m_I

The elementary inertia force dI due to the water particle hori-

zontal acceleration acting on an elementary buoy section of

volume

$$dV = \frac{\pi}{4} d(r)^2 dr$$

is of the form:

$$dI = C_m \rho \, dV \, \ddot{\xi} \cos \theta$$

or, for small angles Θ,

$$dI = C_m \rho \frac{\pi}{4} d(r)^2 \ddot{\xi} dr$$

where C_m = coefficient of added mass for cylinders. This

elementary force can be more simply written:

$$dI = \gamma \, \ddot{\xi} \, d(r)^2 dr$$

with $\qquad \gamma = \frac{\pi}{4} \rho \, C_m$ $\qquad\qquad$ (2.2.35)
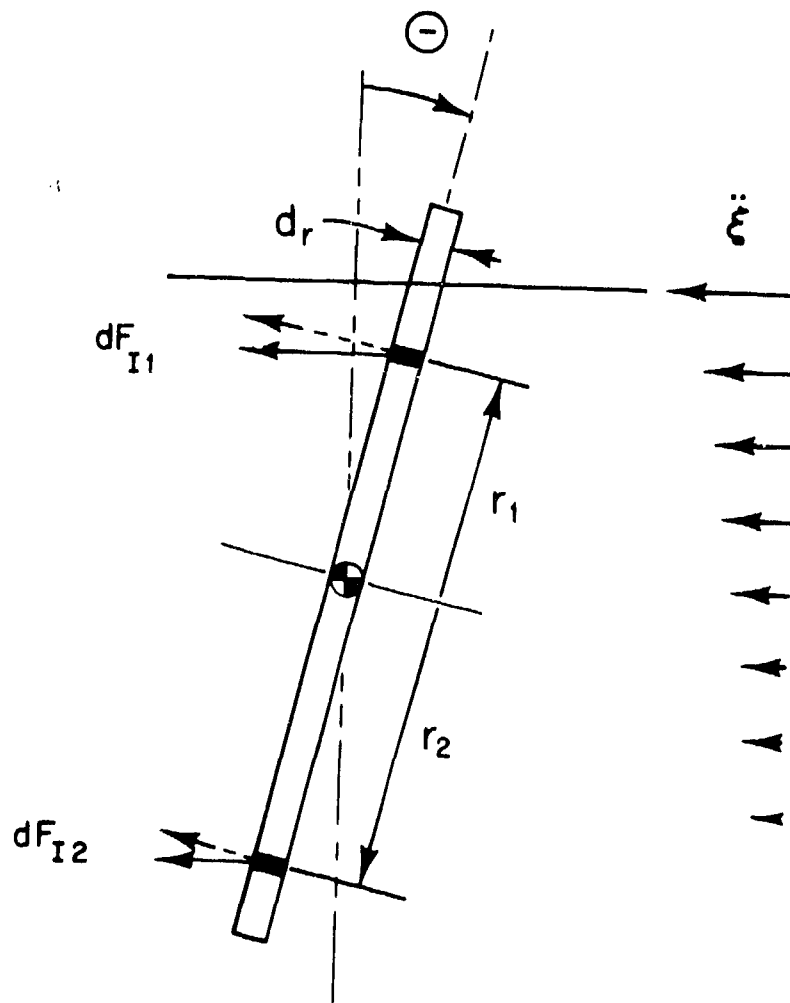
Fig. No. 7

The moment with respect to the c.g. of this elementary force is

in turn given by

$$dM_I = \gamma \, d(r)^2 \, r \, \ddot{\xi} \, dr$$
$$= \gamma \, \omega^2 A \sin \omega t \, d(r)^2 \, r \, e^{-kz} \, dr$$

Nothing again (Fig. No. 7) that inertia forces due to the wave

action have a tendency to upright the buoy above the c.g. and to

capsize it below the c.g., the expression of the total moment

will be given by:

$$M_I = \rho \omega^2 A \sin \omega t \left\{ \int_{r_1=0}^{r_1=S} d(r_1)^2 r_1 e^{-Kz} dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)^2 r_2 e^{-Kz} dr_2 \right\}$$

(2.2.36)

or,

$$M_I = PA \omega^2 \sin \omega t$$

(2.2.37)

A method for the evaluation of the coefficient P is outlined

in Appendix IV.

2.2.2.4   Expression of the added moment of inertia I_F

The added mass of an elementary buoy section of volume

dV, located at a distance r from the buoy c.g. is given by

$$dm' = C_m \rho_F dV$$

C_m = added mass coefficient = 1 for cylinders.

The moment of inertia of this elementary mass with respect

to the c.g. is:

$$dI_F = r^2 dm' = r^2 \rho_F dV$$

and the total moment of inertia is

$$I_F = \int dI_F = \iiint_{V_F} \rho r^2 dV$$

(2.2.38)

Thus

$$I_F = \text{moment of inertia of the water displaced by the buoy}$$

with respect to the buoy c.g.

I_F can be evaluated following the method outlined in Appendix V.

2.2.2.5 Expression of the differential equation of roll

Summing the moments and applying the angular form of

Newton's law yield:

$$-W\overline{gm}\left(\theta - \frac{\omega^2}{g} A \sin\omega t\right) - B\dot{\theta} + DA \cos\omega t + PA\omega^2 \sin\omega t = (I + I_F)\ddot{\theta}$$

The resulting equation of motion is then:

$$I_v\ddot{\theta} + B\dot{\theta} + C\theta = A\left\{\left(\frac{\omega^2}{g} + P\omega^2\right)\sin\omega t + D\omega\cos\omega t\right\}$$

where $C = W\overline{gm}$ is the roll restoring constant.

The equation of roll motion can be further reduced to:

$$C\theta + B\dot{\theta} + I_v\ddot{\theta} = M\cos(\omega t + \tau) \tag{2.2.39}$$

where M, the exciting torque due to wave action, is

given by:

$$M = A\sqrt{\left(\frac{C\omega^2}{g} + P\omega^2\right)^2 + (D\omega)^2} \tag{2.2.40}$$

and τ the phase angle between wave and resulting torque is

in turn given by:

$$\tau = -\tan^{-1}\frac{\dfrac{C\omega^2}{g} + P\omega^2}{D\omega} \tag{2.2.41}$$

2.2.2.6   Expression of the roll response amplitude operator

Assuming again that a particular solution of the differential

equation of roll is given by:

$$\theta = \theta_o \cos (\omega t + \psi)$$

where ψ is the phase angle between roll and the external

torque $\underline{M}$ and introducing this value of θ and the values of

its first and second derivatives $\overset{o}{\theta}$ and $\overset{oo}{\theta}$ in the equation of

motion, will yield

$$\theta = \frac{\underline{M}}{\sqrt{(C - I_V \omega^2)^2 + (\omega B)^2}}$$

and

$$\psi = \tan^{-1} \frac{-\omega B}{C - I_V \omega^2} \qquad (2.2.42)$$

The expression of the roll response will then be given

by:

$$\theta = \frac{A \sqrt{\left(\frac{C\omega^2}{g} - P\omega^2\right)^2 + (D\omega)^2}}{\sqrt{(C - I_V \omega^2)^2 + (\omega B)^2}} \qquad (2.2.43)$$

The response amplitude operator being the ratio of the

roll amplitude by the wave amplitude A , will thus be given

by:

$$R.A.O. = \sqrt{\frac{\left(\frac{C\omega^2}{g} + P\omega^2\right)^2 + (D\omega)^2}{(C - I_V \omega^2)^2 + (\omega B)^2}} \qquad (2.2.44)$$

2.2.2.7 Phase relationship between roll and wave

As previously established, the phase angle σ between wave and exciting torque is given by:

$$\sigma = \tan^{-1}\left[-\frac{\frac{C\omega^2}{g} + P\omega^2}{D\omega}\right] \qquad (2.2.45)$$

The phase angle ψ between the exciting torque and the roll response is in turn given by:

$$\psi = \tan^{-1}\left[-\frac{\omega B}{C - I_V\omega^2}\right] \qquad (2.2.46)$$

Finally the phase angle Φ between wave and roll response will be the sum of the two, i.e.,

$$\Phi = \sigma + \psi \qquad (2.2.47)$$

3.0 COMPUTER PROGRAMS

3.1 Heave Computer Program. (HERAO)

3.1.1 Program logic

The operations performed by the heave computer program can be summarized as follows:

- It computes the heave Response Amplitude Operator, using formula (2.2.23) for decreasing values of the wave angular frequency ω .

The recurrence formula used to change the value of the
angular frequency between two consecutive computations
of the RAO is

$$\omega_n = \frac{2\pi}{T_{n-1} + \Delta T}$$

where ΔT, the change in wave period is constant.
The value of ΔT to use for a particular set of compu-
tations is left as a program input, and so is the range
of variation of wave periods to be considered.

- It computes the phase angles between force and wave,
  heave and force, and heave and wave using formulas
  (2.2.24), (2.2.25), and (2.2.26), for the same set of
  angular frequencies $\{\omega_n\}$.

- It computes the wave amplitudes spectral density using
  one of the spectral density formulas (2.1.11), (2.1.12),
  or (2.1.13) for the same set $\{\omega_n\}$. The choice of spectral
  density formula is left as an input.

- It computes the heave response spectral density $R(\omega)$
  using formula (2.1.8) and the computed values of the
  RAO for the selected set $\{\omega_n\}$.

- It computes the root mean square values of the wave
  amplitudes and of the heave response amplitudes by
  taking, as suggested by formulas (2.1.5) and (2.1.9),
  the square root of the area under the wave and heave
  amplitudes spectral density curves established for the
  set $\{\omega_n\}$

- It uses the statistical results of Tables I and II and the
two root mean square values of wave amplitudes and
heave response to compute the corresponding expecta-
tions of wave and heave means and maxima.

3.1.2   Program input

The program is designed to handle any number of cases in
consecutive order. All input is format free. Values for any
parameters are entered in a continuous string, separated by
commas. The program is designed to run either in the batch
mode or interactively from an on-line remote terminal. The
method of input is the same for either case. As the interactive
mode is also self-explanatory and types user prompts, the
input will be discussed for the batch mode. All depths are
considered as positive downwards. An equivalent depth is an
average, or more accurately an effective, depth at which a
body or surface is located.

Input data must be provided on the following cards:

Card 1 -- Number of pressure surfaces.

   NP   an integer value, starting in column 1, used to
          specify the number of horizontal pressure
          surfaces.

Card 2 -- Pressure depth, surface area.

          There will be as many card 2's as specified on
          card 1. Each will contain the following informa-
          tion.

DEPTHP   a value specifying the "equivalent depth"

of the ith pressure surface, in feet.

AREA   a value specifying the area of the pressure sur-

face, in square feet. A negative value is entered

for a surface that has exerted upon it a downward

force. A positive value is entered for surface

subjected to an upward force.

Card 3 -- Number of inertial components.

NE   an integer value, starting in column 1, specifying

the number of inertial components which comprise

the buoy.

Card 4 -- Depth, added mass coefficient, volume.

There will be as many card 4's as specified on

card 3. Each card will contain the following infor-

mation.

DEPTHI   a value specifying the equivalent depth of the ith

inertial component, in feet.

ADDMSC   a value specifying the added mass coefficient

for the ith inertial component.

VOLUME   a value specifying the volume of the ith inertial

component, in cubic feet.

Card 5 -- Number of drag surfaces.

ND   an integer value, starting in column 1, specifying

the number of drag surfaces of the buoy body.

Card 6 -- Drag depth, linearized damping coefficient,

linearized wave drag coefficient.

There will be as many card 6's as there are drag

surfaces specified on card 5. Each card will con-

tain the following information.

DEPTHD a value specifying the equivalent depth, in

feet, of the ith drag surface.

DAMPC a value specifying the linearized damping

coefficient of the ith drag surface, in lbs

force/(ft/sec)/(rad/sec).

WDRAGC a value specifying the linearized wave drag

coefficient of the ith drag surface, in lbs

force/(ft/sec)/(rad/sec).

Card 7 -- Cross sectional area at water surface.

CAREAWL a value, starting in column 1, specifying the

cross sectional area at the water line, in square

feet. For the purposes of this analysis this area

is assumed to be constant over the range of motion

at the water line.

Card 8 -- Virtual mass.

VIRTMASS a value specifying the virtual mass of the

body, in slugs.

Card 9 (3F.0) -- Starting, ending, increments of wave periods.

TIME1 a value specifying the lowest wave period to be

studied, in seconds.

TIME2 a value specifying the highest wave period to be

studied, in seconds. This time should be an in-

tegral multiple of the incremental time (TIMEDEL)

greater than TIME1.

TIMEDEL a value specifying the incremental wave period,

in seconds, used in the analysis from TIME1 to

TIME2.

Card 10 (I, F.0, F.0) -- Amplitude spectrum selection, para-

meters.

This card has a general form as follows.

ISEASEL, PARAMA, PARAMB

The necessity and meaning of the parameters will

depend on the amplitude spectrum (ISEASEL)

selected. In reality these are double height formulas

which are converted internally to give the amplitude

spectrum. The following options are available.

ISEASEL = 1 Pierson-Moskowitz formula.

PARAMA = wind speed, in knots

no PARAMB

ISEASEL = 2 Bretschneider formula.

PARAMA = significant wave height, in feet

PARAMB = significant wave period, in seconds

ISEASEL = 3 International Ship Structure Congress

PARAMA = significant wave height, in feet

PARAMB = significant wave period, in feet

Card 11 (A1, 1X, A1, 1X, 2E. 0) -- Selection of listing, line printer

plot, plot scale minimum, plot scale maximum.

ILIST enter a Y (for yes) in column 1 if you desire a

listing of the various output parameters. Any

other character in column 1 will not produce a

listing.

IPLOT enter a Y (for yes) in column 3 if you desire a

line printer plot of the RAO. Any other char-

acter in column 3 will not produce a plot. The

line printer will plot a point at each selected wave

period. A check is made on the length of the plot

for the following criteria.

$$N = \frac{T2 - T1 + TIMEDEL}{TIMEDEL} \leq 250$$

Note: This limitation is computer dependent.

RAOMIN if a plot is desired, you may enter, beginning

in column 5, the minimum value for the RAO

scale. If left blank, RAOMIN = 0.

RAOMAX if a plot was selected, the maximum value of

the RAO scale may be entered following RAOMIN

(separated by a comma). Under the current

version RAOMAX $\geq$ 5. For best results RAOMAX-

RAOMIN should be an integral multiple of 5. If

it is not, the program adjusts it to be so. The
current default is RAOMAX = 5.

Card 12 (A1) -- Another case?

IEND   If you wish to run another case, enter a Y or
YES beginning in column 1. Any character other
than a Y in column 1 will cause the program to
terminate.

The sequence of card types 1 through 12 is repeated for
each additional case desired.

There is a special entry mode for additional cases. Be-
cause the buoy configuration may be quite complex, it is un-
desireable to enter all the descriptive parameters if all that is
changing is the wind speed for the sea state. Another alternative
is that all the parameters may remain constant except the inertia
terms. A special input code of -1 will allow the user to keep in
effect the values last entered for any of the parameters. This
input code may be used for any of the following input cards.

Card 1

NI = -1

Use the pressure parameters from the previous case.
Do not input any type 2 cards.

Card 3

NI = -1

Use the inertia parameters from the previous case.
Do not input any type 4 cards.

Card 5

ND = -1

Use the drag parameters from the previous case.

Do not input any type 6 cards.

Card 7

CAREAWL = -1

Use the previously entered value of the cross sectional

area.

Card 8

VIRTMASS = -1

Use the previously entered value of the virtual mass.

Card 9

TIME 1 = -1

Use the previously entered time range and increments.

Card 10

ISEASEL = -1

Use the previously entered amplitude spectrum for the

sea state.

Usage modes

As previously mentioned, the program can be used in either

a batch or an interactive mode.

The control card sequence necessary to compile, load, and

run the HERAO program in a batch mode is as follows:

```
!JOB aaa,uuu
!LIMIT (TIME,3),(CORE,10)
!FORTRAN LS,GO
```

FORTRAN source deck of program HERAO

!METASYM SI, LO, GO

source deck for subprogram IAMTERM

!LOAD (GO), (UNSAT), (3)), (MAP), (LDEF), (LMN, HERAOR),
  (PERM)
!RUN (LMN, HERAOR)
!DATA

data cards for each case

!EOD

To run a subsequent job utilizing the existing load module,

only the following cards need to be submitted:

!JOB aaa, uuu
!LIMIT (TIME, 2), (CORE, 10)
!RUN (LMN, HERAOR)
!DATA

data cards for each case

!EOD

To use the run module in an interactive mode from a terminal,

simply log on and enter, to a ! prompt, the following:

START HERAOR

where HERAOR is the name of a previously created load module.

From this point, the operator simply responds to the pro-

gram prompts as if you were punching up the cards. The only

difference is that the operator does not need to start in column 1,

but should start as though the head were already positioned cor-

rectly. It is.

Restrictions

The user should not enter a wave period of zero (0.0) or less. While the program will still run, the integral of the wave amplitudes from an angular frequency of ∞ will be unreasonable. This in turn will cause the resulting wave statistics to be in error. All other parameters computed should be satisfactory. The method of integration used is that of a trapezoidal approximation. The user must therefore exercise some care in selecting the time period increment. Too large an increment may cause the peak of the RAO or wave amplitude spectrums to be "smoother", resulting in lower values for the integrals of the he. e response and amplitude spectrum.

Subprograms required

IAMTERM   a metasymbol subprogram which checks to see if the program is being run in batch or from an on-line terminal.

SEASPEC   computes the double height density spectrum for the sea state according to one of several empirical formulas; internal.

PLOTINIT   initializes the line printer plot routine; internal.

PLOTHEAV   executes the line printer plot routine; internal.

LPPLOT (PLOT1, PLOT2, PLOT3, PLOT4, PLOT5, PLOT7)

     a subprogram which helps create and list a line printer plot; from W.H.O.I. account 3 library.

3. 1. 3   Program output

The output of the program is comprised of four basic parts. These are:

1. Summary of input parameters.

2. Summary of RAO, phases, and amplitude spectrum, all given as a function of time and frequency.

3. Tabular summaries of wave and heave response statistical properties.

4. Line printer plot of the RAO.

The summary of the input parameters is only given for a run made in the batch mode. For an on-line hard copy terminal, the users entries constitute the input summary.

The listing of the RAO, phases, and other information is optional, as specified in column 1 of input card type 11. The list has the same format whether in the batch or on-line mode. Note that the amplitude spectrum is output for the sea state.

Tabular summaries of the wave and heave response statistical properties are always output and are the same regardless of the mode of operation.

The line printer plot is also optional, as specified in column 3 of input card type 11.

Typical program outputs for the batch and terminal modes are shown under "Case Studies", Section 4.

Errors and diagnostics

***NUMBER OF ENTRIES IS GREATER THAN ARRAY

SIZE ALLOWS

$$nn_1 \qquad\qquad nn_2$$

THE PROGRAM TERMINATES

The input for the number of components describing the buoy

configuration exceeds the array size allocated. Currently $nn_2 = 20$.

***THE PLOT BUFFER IS NOT LARGE ENGOUGH FOR

THE PERIOD RANGE SPECIFIED

THE PLOT IS SUPPRESSED

The number of wave periods analyzed must meet constraints

described in the input section, card 11.

Timing

The program's execution time is a function of the buoy con-

figuration and the number of wave periods analyzed. In any case,

the execution time normally is negligible, being about 3 seconds

(0.05 minutes) per case.

3.2 Roll Computer Program (ROLLRAO)

3.2.1 Program logic

The operations performed by the roll computer program

are similar to those performed by the heave program. They

include:

- Computation of the roll RAO, using formula (2.2.44)

over the set $\{\omega_n\}$ previously defined. The roll RAO

is expressed in units of degrees of roll per foot of wave amplitude. The recurrence formula to change the value of the angular frequency between two consecutive computations of the RAO is again

$$\omega_n = \frac{2\pi}{T_{n-1} + \Delta T}$$

where ΔT, the change in the wave period is a constant set by the user.

- Computation of the phase angles between external torque and wave, roll response and torque, and roll and wave using formulas (2.2.45, 46, 47) for the same set $\{\omega_n\}$.

- Computation of the wave amplitude and roll response spectral densities $S(\omega)$ and $R(\omega)$ and of the root mean square values of wave amplitudes and roll amplitudes. The choice of spectral density formula is left as a program input.

- Finally, computations of expectations of means and maxima of wave and roll amplitudes with the help of the statistical parameters shown in Tables I and II.

3.2.2 Program input

The program is designed to handle any number of cases. Almost all input is format free. Values for any parameter are entered in a continuous string, separated by commas or blanks. The program is designed to run either in the batch mode or interactively from an on-line remote terminal. As the interactive

mode is also self-explanatory and types user prompts, the
input will be discussed for the batch mode. All depths are
considered as positive downwards.

Card 1 (3F,0) -- Period range of time.

TIME1 a value specifying the lowest wave period to
be studied, in seconds. As the wave velocity
expression contains an exponent with wave fre-
quency, the user is cautioned against usi..g a
starting period of less than 1.0 seconds.

TIME2 a value specifying the highest wave period to be
studied, in seconds. This value of time should
be an integral multiple of the incremental time
(TIMEDEL) greater than the value of TIME1.

TIMEDEL a value specifying the incremental wave
period, in seconds, used in the analysis from
TIME1 to TIME2.

Card 2 (I,F.0,F.0) -- Amplitude spectrum selection, parameters.
This card has a general form as follows.

ISEASEL, PARAMA, PARAMB

The necessity and meaning of the parameters will depend
on the amplitude spectrum (ISEASEL) selected. In
reality, these are double height formulas which are
converted internally to produce the amplitude spectrum.
the following options are available.

ISEASEL = 1   Pierson-Moskowitz formula

    PARAMA   wind speed, in knots.

  no PARAMB

ISEASEL = 2   Bretschneider formula.

    PARAMA   significant wave height, in feet.

    PARAMB   significant wave period, in seconds.

ISEASEL = 3   International Ship Structure Congress

    PARAMA   significant wave height, in feet.

    PARAMB   significant wave period, in seconds.

Card 3 (F.0) -- Radius of buoy at water surface plane.

  RWL   a value specifying the outer radius of the

        buoy at the surface (still water assumed), in feet.

Card 4 (F.0) -- Depth to keel.

  DEPTHK   a value specifying the depth to the keel (bottom)

        of the buoy, in feet.

Card 5 (F.0) -- Average wave amplitude.

  AVERGAMP a value specifying the average expected

        wave amplitude, in feet.

Card 6 (F.0) -- Average roll constant.

  THETABAR a value specifying the average expected

        roll, in degrees.

Card 7 (I) -- Number of buoy components.

  NP   a value specifying the number of buoy com-

        ponents. This may be set to zero in subsequent

        cases.

<u>Cards 8 ... (7+NP)</u> (I, 6F. 0, 3A4, A2) -- Component specification.

ISHAPE   an integer value used as a code to specify

the component shape.

1) hollow cylinder

2) solid cylinder

3) solid disc

4) right triangular plate

WIDTH   a value specifying the width (diameter or

base) of the component, in feet.

HEIGHT   a value specifying the height of the component,

in feet.

THICK   a value specifying the thickness of the com-

ponent, in inches. Entering THICK = -1 for

ISHAPE = 1 will generate a solid (THICK =

WIDTH/2. 0). For ISHAPE = 2 or 3, also

enter a -1 as it is ignored.

DENSITY   a value specifying the density of the com-

ponent, in pounds mass per cubic foot

$(lbsm/ft^3)$.

DISTCG   a value specifying the vertical distance from

the buoy keel to the component center of

gravity, in feet. The component c.g. is

vertical vector only.

FRACNORM   a value specifying the fractional propor-

tion of the component area normal to the roll

motion. For cylinders this entry = 1.0. For

triangular plates it will vary from 0.0 (oriented

in line with roll) to 1.0 (area normal to roll

motion).

ICOMMENT   a character string used to describe the

component.

Note: For the purpose of visually inspecting data cards used

in batch input, it may be desirable to "format" the

data. The recommended format is (I5, 6F10.0, 1X,

3A4, A2).

Card 9 (I) -- Redefined part code.

N   an integer value specifying the number (index)

of the component to be redefined. This allows

the user to change the dimensions of a com-

ponent(s) or to add new components. To

add a new component, the specified value of

N must be one greater than the current maxi-

mum number of parts defined for the buoy.

There may be as many redefinition pairs of

cards as desired. To terminate the sequence

enter a value for N = -1.

Card 10 (I, 6F. 0, 3A4, A2) -- Redefinition specification.

This card has the same format as card 7, the component

specification. Any number of pairs of this and the pre-

vious card may be entered as desired.

Card 11 (A1, 1X, A1, 1X, 2F. 0) -- Selection of listing, line

printer plot, plot scale minimum and maximum.

ILIST enter a Y (for yes) in column 1 if you desire

a listing of the various output parameters.

Any other character in column 1 will cause

the listing to be suppressed.

IPLOT enter a Y (for yes) in column 3 if you desire

a line printer plot of the roll RAO. Any other

character in column 3 will cause the line

printer plot to be suppressed. The line

printer plots a point at each selected wave

period. A check is made on the length of the

plot for the following criteria.

$$\frac{T2 - T1 + TIMEDEL}{TIMEDEL} \leq 100$$

RAOMIN if a plot is desired, you may enter, begin-

ning in column 5, the minimum value for the

RAO scale. If left blank RAOMIN = 0.0.

RAOMAX if a plot was selected, the maximum value

of the RAO scale may be entered following

the RAOMIN value (separated by a blank or

comma). For the best results RAOMAX -

RAOMIN should be an integral multiple of

5. If it is not, the program adjusts it to be

so. The current default is RAOMAX = 10.0.

Card 12 (A1) -- Another case?

    IEND   If you wish to run another case, enter a Y or

                   YES beginning in column 1. Any character

                   other than a Y in column 1 will cause the pro-

                   gram to terminate.

The sequence of card types 1 through 12 is repeated for each additional case desired.

There is a special entry mode for additional cases. Because the buoy configuration may be quite complex, it is undesirable to enter all the descriptive parameters to investigate the effect of a different wind speed or sea state. Another alternative is that the design of the buoy remains the same except for the size of the counterweight. A special input code of -1 and the component redefinition options allow the user to keep in effect the values last entered for any of the parameters or components.

    Cards 1-7   A -1 will maintain those values previously

                   entered. For card type 7, this simply uses

                   the same buoy and no new components are

                   defined (no type 8 cards).

    Cards 9-10   As many of these pairs as desired may be

                   entered to define a new buoy configuration.

                   The entry sequence is terminated by a -1

for card type 9 and no card type 10.

Usage

As previously mentioned, this program is meant to be
used in either a batch or an interactive mode. The following
control card sequence is a complete list of steps necessary to
compile, load, and execute the program ROLLRAO in the
batch mode.

```
!JOB aaa, uuuu
!LIMIT (TIME, 3), (CORE, 20)
!FORTRAN LS, GO
   Fortran source deck of program ROLLRAO
!METASYM SI, LO, GO
   source deck of subprogram IAMTERM
!LOAD (GO), (UNSAT, (3)), (MAP), (LDEF), (LMN, ROLLR), (PERM)
!RUN (LMN, ROLLR)
!DATA
   data cards for all cases
!EOD
```

To run a subsequent job utilizing the existing load module,
only the following cards need to be submitted:

```
!JOB aaa, uuuu
!LIMIT (TIME, 2), (CORE, 20)
!RUN (LMN, ROLLR)
!DATA
   data cards for all cases
!EOD
```

To run the load module from a terminal, simply log on and
enter, to a ! prompt, the following:

START ROLLR

From this point on you simply respond to the program
prompts as if you were punching the data cards. The only dif-
ference is that you do not need to start in column 1, but should

start as if you are already there. (There is also a slight difference in the manner in which the buoy components are described and entered. This is explained to the user on line.)

Restrictions

The user should exercise care in entering wave periods of less than 1.0 seconds. The water velocity terms contain a natural exponent of the wave number times displacement from the center of gravity and this computation may exceed the machine computational capability.

The method of integration used is that of a trapezoidal approximation over frequency. The user must therefore exercise additional care in selecting the time period increment. Too large an increment may cause the peak of the RAO or wave amplitude spectrums to be "smoothed", resulting in lower values for the integrals of the amplitude spectrum and roll response. At the other extreme, the time period of 0.0 will cause an "infinite" angular frequency and the resulting wave statistics will be in error.

Also note that the entire program is executed in single precision. For normal buoy configurations this mode is adequate. However, in certain cases such as the case of a small flat cylinder (case study no. 1, Section 4.1), the response amplitude operator exhibited signs of instability in the numerical computation. This can be overcome by using double precision

computations.

Subprograms Required

IAMTERM	a Metasymbol subprogram which checks to see if the program is being run in batch or from an on-line terminal.
SEASPEC	computes the amplitude density spectrum for the sea state according to one of several empirical formulae; internal.
PLOTINIT	initializes the line printer plot routine; internal.
PLOTROLL	executes the line printer plot routine; internal.
TINPUT	prompts and inputs data from a user on-line.
BINPUT	reads and summarizes data entered in batch mode.
BODYVOL	computes the volume of a component.
BODYMI	computes the shape dependent contribution of a component's moment of inertia.
DISPLACE	computes those parameters associated with the displacement of the buoy's components.
BUOYDAMP (WATERDAMP)	computes the moments of damping for the buoy and water drag forces.
WATRINRT	computes the inertia moment contribution from the water wave particle acceleration.

LPPLOT (PLOT1, PLOT2, PLOT3, PLOT4, PLOT5, PLOT7)

a subprogram which creates and tests a

line printer plot; W.H.O.I. account 3

library.

3.2.3   Program output

The output of the program is comprised of four basic

parts. These are:

1. Summary of input parameters and physical properties

    of the buoy.

2. Summary of RAO, phases, and amplitude spectrum,

    all given as a function of time frequency.

3. Tabular summaries of wave and roll response statis-

    tical properties.

4. Line printer plot of the roll RAO.

The summary of the input parameters is only given for a

run made in the batch mode. For an on-line hard copy terminal,

the user entries constitute the input summary.

The listing of the RAO, phases and other information is

optional, as specified in column 1 on input card type 11. The

list has the same format whether in batch or on-line mode. Note

that the amplitude spectrum is output for the sea state. This

is derived directly from the selected double height spectrum.

Tabular summaries of the wave and roll response statis-

tical properties are always output and are the same regardless

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of the mode of operation.

The line printer plot is also optional, as specified in column 3 of the input card 11.

Timing

The program's execution time is a function of the buoy configuration and the number of wave periods analyzed. Normally the execution time is much less than a minute per case.

Errors and Diagnostics

A number of checks are made on the input parameters and the stability of the buoy. The messages indicate the nature of the error and take appropriate action depending on the execution mode.

4.0 CASE STUDIES

To illustrate the use of the computer solutions the following three case studies are presented.

    a. Heave and roll response of a flat cylinder of small dimensions.

    b. Heave and roll response of a ballasted "telephone pole."

    c. Heave and roll response of a complex shape W.H.O.I. spar buoy.

4.1 Heave and Roll Response of a Small Flat Cylinder

This case study has relatively little practical value. Every one knows that a thin slice of pulpwood, if thrown in the sea, will essentially follow the heave and slope of the waves, both in magnitude and phase. It is included here mainly for the

purpose of program result verification.

The small cylinder considered has a diameter of one foot. The ratio of its height to its diameter is 1:3. Its density is 3/4 that of sea water. The heave and roll motion of this cylinder will be studied using a Pierson-Moskowitz spectrum with a 20 knots wind.

4.1.1 Program input

Buoy draft. The buoy draft is obviously 0.25'.

Pressure surface. The pressure force is exerted exclusively on the cylinder lower face. The pressure surface is therefore at 0.25' from the surface. Its area S is $\pi/4 = 0.785$ sq. ft. The pressure force being upwards S is positive.

Inertial component. The added mass effect will be con-considered to take place at the lower face of the cylinder, i.e. at 0.25' from the surface. It will be assumed to take place only half of the time, during the downwards part of the heave cycle. It will be estimated to be the same as the one produced by a flat plate of same radius as the cylinder. Thus the averaged added mass will be:

$$m' = \frac{1}{2}\left(\frac{8}{3}\rho a^3\right) = \frac{1}{2}\left(C_m \rho Vol\right) \approx 0.33 \text{ slug.}$$

Assuming the added mass coefficient to be equal to one, the averaged added mass coefficient and volume will be

$$C_M' = \frac{1}{2} C_M = 0.50$$

$$V_L = \frac{8}{3} a^3 = 0.33 \text{ cu-ft.}$$

Damping/drag surface. Damping and wave drag will also be considered to take place at the lower face of the cylinder, i.e. at 0.25' from the surface. These effects will again be assumed to take place only half the time.

A choice must now be made for the arbitrary values of average heave $\overline{X_b}$ and average wave amplitude $\overline{X_c}$. Assuming that the heave equals the wave amplitude, a fair assumption in this case, and selecting the average wave amplitude for winds of 20 knots to be 3' will yield $\overline{X_b} = \overline{X_c} = 3'.$

Using this value of $\overline{X_b}$ and $\overline{X_c}$ in expressions (2.2.10) and (2.2.13), and a drag coefficient $C_D = 0.9$, the value of the linearized damping and wave drag coefficients b' and c' is found to be:

$$b_1' = c_1' = 1.8 \frac{lb_-^{\rho}}{ft\text{-}sec} \Big/ \frac{rad.}{sec}$$

To account for the time average only half of these coefficients value, i.e. 0.9, should be used as program input.

Cross sectional area at surface. This area equals $\pi/4$ or 0.785 sq. ft.

Buoy virtual mass m_v

$$M_v = M + M'$$

M = buoy mass = $\frac{\pi}{4} \times \frac{1}{3} \times \frac{3}{4} \times \frac{64}{32}$ = 0.392 slug.

M' = added mass = <u>0.333 slug</u>

Thus, buoy virtual mass m_v = 0.725 slug

In addition to these parameters, the user must also specify the arbitrary roll constant $\overline{\Theta}$ and wave amplitude constant $\overline{A}$. In this case the average angle roll $\overline{\Theta}$ will be assumed to equal the slope of the average wave in 20 knots wind. Assuming an average amplitude $\overline{A}$ of 3' and an average wave length of 120', the average wave slope $\overline{\phi}_s$ is found to be:

$$\overline{\phi}_s = \overline{\Theta} = \frac{2\pi\overline{A}}{L} = \frac{\pi}{20} \qquad = 0.157 \text{ radians}$$

A summary of the data input is shown in the Data Coding Form Fig. No. 8 and Fig. No. 9.

4.1.2 Program output

Heave and roll response amplitude operators are depicted in Fig. Nos. 10 and 11.

Computed expectations of average and maximum values of wave amplitudes and of heave and roll motions are also obtained with the help of the computer programs. Corresponding values of wave slopes are calculated independently using

$$\overline{\phi} = \alpha \sqrt{\int \phi(\omega)\, d\omega}$$

where α is the applicable Raleigh constant

and $\phi(\omega) = K^2 S(\omega)$,

K being the wave number $= \omega^2/g$ and $S(\omega)$ being the wave amplitude spectral density.

These results are summarized in Table Nos. 3 and 4.

The heave RAO, with a value of one over most of the

DATA CODING FORM

DATA FOR THE HEAVE MOTION OF A SMALL, FLAT CYLINDER (CASE #1)

```
1
0.25,  0.785
1
0.25,  0.50,  0.33
1
0.25,  1.80,  1.80
0.785
0.725
1.0,  50.0,  1.0
1,  20.0
Y, Y
N
```

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Fig. No. 8

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DATA CODING FORM

DATA FOR THE ROLL MOTION OF A SMALL FLAT CYLINDER (CASE #1)

```
 1.  0,  50.0,  1.0
 2.  , 20.0
 3.  0.50
 4.  0.25
 5.  3.0
 6.  9.0
 7.  -1
 8.  2., 1.0, 0.333, -1.0, 48.0, 0.1, 667., 1.0, 0.1  FLAT DISC
 9.  -1
10.  Y, Y
11.  N
```

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Fig. No. 9

Fig. No. 10
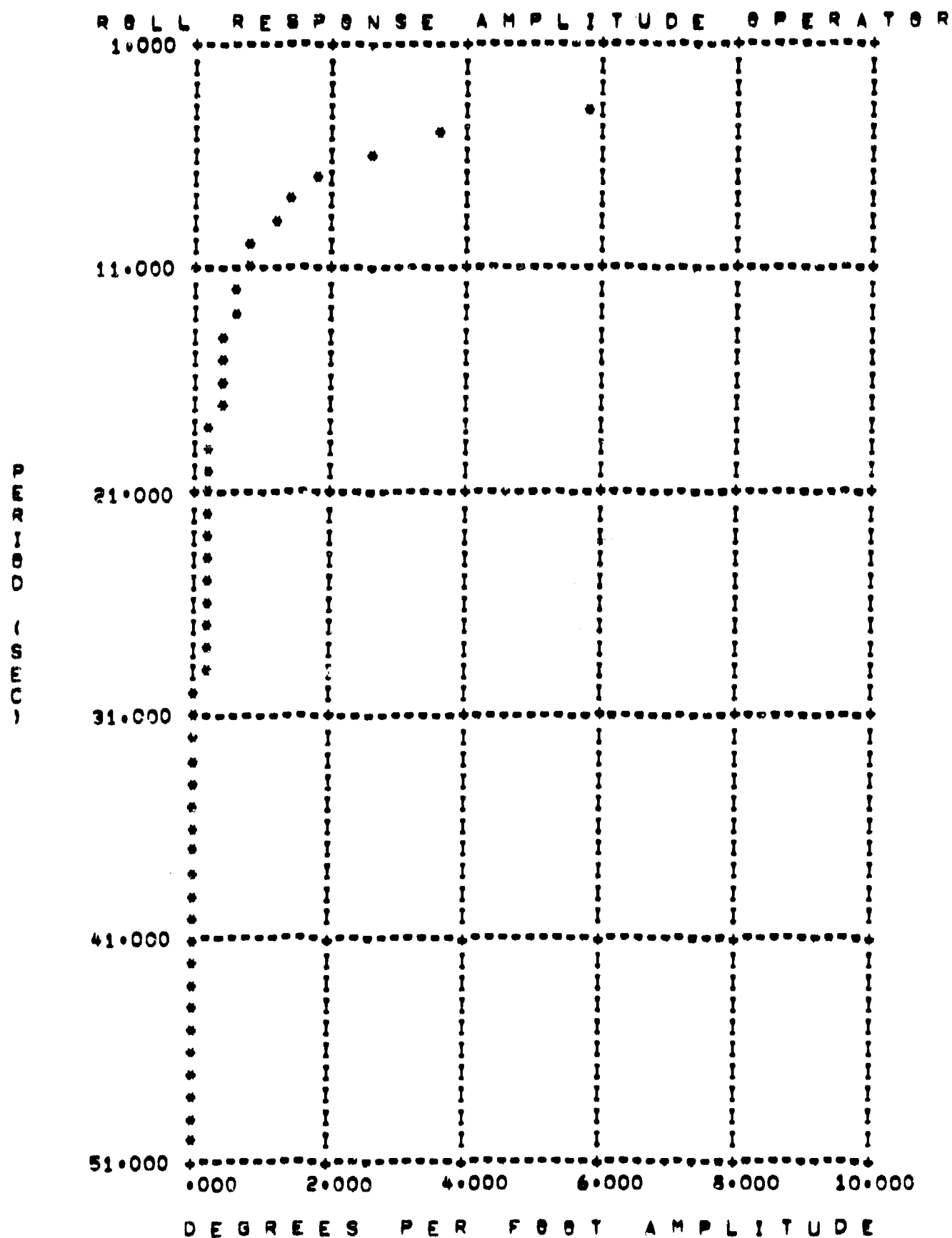
Fig. No. 11

RMS OF WAVE SPECTRUM = 2.683 FEET

PROBABLE AMPLITUDE
OF WAVE = 1.897 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE WAVE AMPLITUDE		NUMBER OF WAVES	EXPECTED WAVE MAXIMUM AMPLITUDE
.010	6.330	I	50	5.689
.100	4.830	I	100	6.118
.333	3.799	I	500	7.003
.500	3.370	I	1000	7.459
1.000	2.377	I	10000	8.399
		I	100000	9.311

RMS OF RESPONSE SPECTRUM = 2.684 FEET

PROBABLE AMPLITUDE
OF HEAVE RESPONSE = 1.897 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE HEAVE AMPLITUDE RESPONSE		NUMBER OF WAVES	EXPECTED HEAVE MAXIMUM AMPLITUDE
.010	6.331	I	50	5.690
.100	4.831	I	100	6.119
.333	3.800	I	500	7.005
.500	3.371	I	1000	7.461
1.000	2.378	I	10000	8.400
		I	100000	9.313

Table No. 3   Heave Response of Flat Small Cylinder

Amplitudes of Roll are in Degrees

Fraction of Largest Amplitudes Considered	Average Roll Amplitude Response	I	Number of Waves	Expected Roll Maximum Amplitude
		I		
.010	25.070	I	50	22.530
.100	19.129	I	100	24.230
.333	15.048	I	500	27.737
.500	13.348	I	1000	29.544
1.000	9.416	I	10000	33.264
		I	100000	36.877

Amplitudes of Wave Slope are in Degrees

Fraction of Largest Amplitudes Considered	Average Slope	I	Number of Waves	Expected Maximum Slope
		I		
.010	23.905	I	50	21.483
.100	18.240	I	100	23.104
.333	14.349	I	500	26.448
.500	12.728	I	1000	28.171
1.000	8.978	I	10000	31.718
		I	100000	35.163

Table No. 4   Roll Response of Small Flat Cylinder

wave periods considered, clearly indicates that the small
cylinder is a perfect wave follower. Averages and expected
maximum values of heave when compared to corresponding
values of wave amplitude confirm this expected result.

The roll RAO, being in degree of roll per foot of wave
rather than per degree of slope does not immediately correlate
roll and wave slope. It shows simply that roll is large at
small periods, and tends to zero as the wave period increases,
which is of course precisely what the slope of the wave does.
The computed statistical averages however do confirm that
for practical purposes the roll angle is strongly correlated to
wave slope. It thus appears that this first study case is a good
test of the program validity.

4.2   Heave and Roll Response of a Ballasted "Telephone Pole"

We next consider a cylindrical body made of two cylinders
of same diameter, but of different lengths and densities, as
shown on Fig. No. 12.

The density of the large cylinder is 16 lbs/cu. ft. The
density of the small cylinder is 384 lbs/cu. ft. As in the previous
case, the heave and roll response of this body will be studied
using a Pierson-Moskowitz spectrum with a 20 knots wind.

4.2.1   Program input

In order to provide the necessary input data to the heave
and roll programs, the following computations must be first
performed.
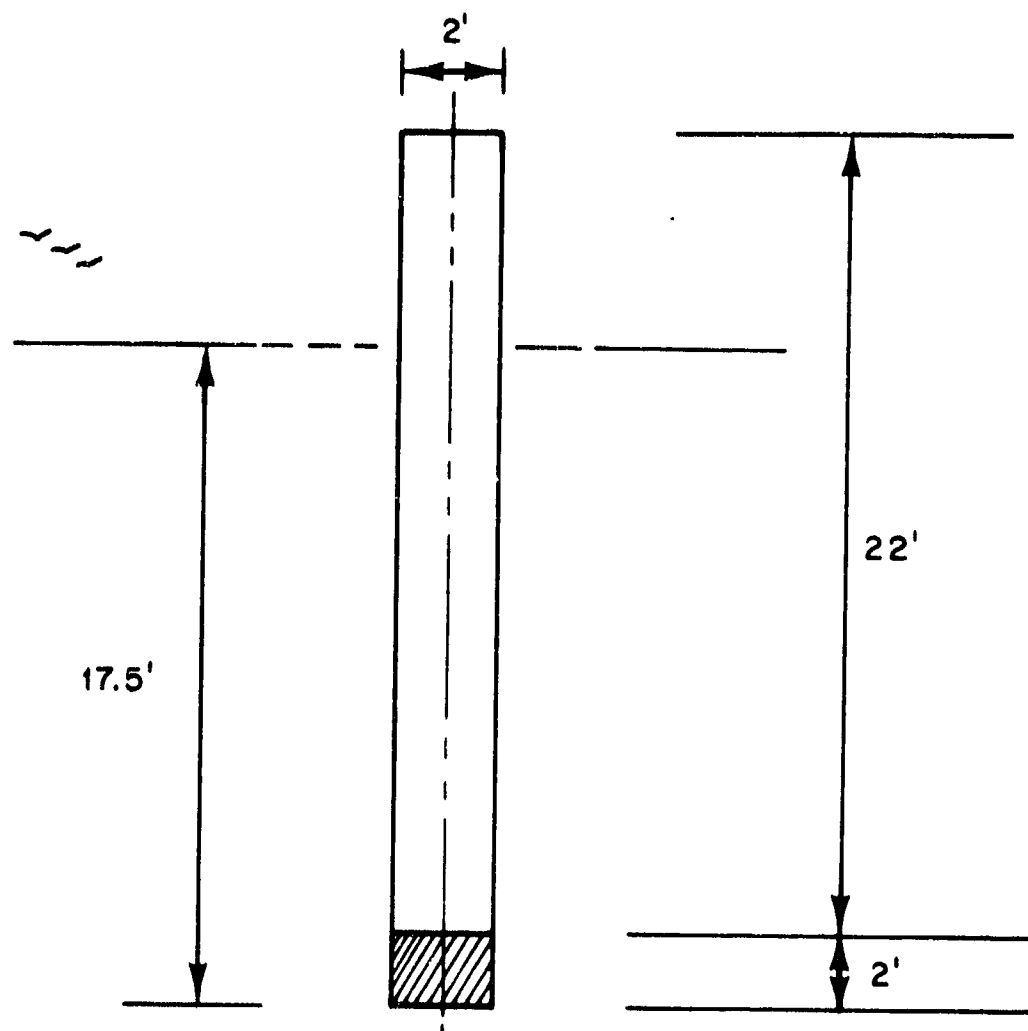
Fig. No. 12

Buoy draft.

weight of small cylinder = π x1x1x2x384 = 2413 lbs

weight of long cylinder = π x1x1x22x16 = 1105 lbs

Total Weight = 3518 lbs

$$\text{Draft} = \frac{3518}{\pi \, x1x1x64} = 17.5'$$

Number, depth, and area of pressure surfaces. In this

simple case there is again only one pressure surface, namely

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the pole lower face. It is located 17.5' from the surface. Its

area "S" is

$$S = \pi \times / r / , \quad S = 3.14 \text{ sq. ft.}$$

The pressure force being upwards, S is positive.

Number, depth, added mass coefficient and volume of

inertial components. In heave motion, the added mass effect

will be considered to be essentially produced by the pole lower

end. There will thus be only one inertial component, acting at

17.5' from the surface. The added mass effect will be estimated

to be the same as the one produced by a sphere of same radius

as the pole (long cylinder approximation), but acting only half

of the time. Thus the averaged added mass will be expressed

by

$$m' = \frac{1}{2} C_m \rho \, Vol = \frac{1}{2}\left(\frac{1}{2}\frac{64}{32.2} \times \frac{4\pi}{3}\right) = 2.08 \quad \text{slugs}$$

The corresponding added mass coefficient and volume

are therefore

$$C_m' = \frac{1}{2} C_m = 0.250$$
$$Vol = \frac{4}{3}\pi = 4.19 \text{ cu. ft.}$$

Number, depth, damping and wave drag coefficients of

drag surfaces. Damping and wave drag effects are assumed to

be also essentially produced by the pole lower face. Thus there

will be only one drag surface located at 17.5'. These effects

will be assumed to take place only half of the time.

The average heave $\overline{X}_b$ will be assumed to be half of the average wave amplitude $\overline{X}_c$ and the later will be assumed to be 3'. Using these values of $\overline{X}_b$ and $\overline{X}_c$ in expressions (2.2.10) and (2.2.13) together with $C_D = 0.9$, yield

$$b_1' = 3.6 \qquad \frac{lb\text{-}f}{ft\text{-}sec}, \quad / \frac{rad.}{sec}$$
$$c_1' = 7.2$$

Again, to account for the fact that damping effects are assumed to occur only half of the time, only half of the co-efficients values, i.e. 1.8 and 3.6 respectively, should be used as program input.

<u>Cross sectional area at surface.</u>

$$\pi R^2 = \pi \times 1 \times 1 = 3.14 \text{ sq-} ft .$$

<u>Buoy virtual mass m_v.</u>

$$M = \frac{3518}{32.2} \quad = \quad 109 \text{ slugs}$$

$$m' = \qquad \qquad \underline{2.08} \text{ slugs}$$

$$m_v = m + m' \quad = \quad 111.08 \text{ slugs}$$

In addition to these computed parameters, the program user must select the arbitrary average roll constant $\overline{\Theta}$ and wave amplitude $\overline{A}$. For this example $\overline{\Theta}$ is set equal to 5° and $\overline{A}$ equal to 3.0 feet.

Obviously the number of buoy parts is two. Their characteristics are summarized as follows:

Part No.	Name	Shape	Width (ft)	Height (ft)	Thick (ft)	Density (lbs/cu.ft)	C. G. Above Keel (ft)
1	Upper Cyl.	Solid Cyl.	2	22		16	13
2	Lower	Solid Cyl.	2	2		384	1

All pertinent input data are listed in the data coding form

shown in Fig. Nos. 13 and 14.

4.2.2   Program output

Values of the heave and roll response amplitude operators

of the telephone pole for the prescribed period interval (50 seconds)

and increment (1 second) are presented in the typical computer

printouts shown in Fig. No. 15 and Fig. No. 16.   The response

amplitude operators are also graphically represented in Fig.

Nos. 17 and 18.   Expected average and maximum values of

wave amplitudes and of heave and roll responses are as tabu-

lated in Tables 5 and 6.

4.3   Heave and Roll Response of a Complex Shape W.H.O.I.

Spar Buoy

Fig. No. 19 shows the dimension and shape of the spar

buoy to be studied next.

Heave and roll response will be again established using a

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DATA CODING FORM

DATA FOR THE HEAVE MOTION OF A BALLASTED POLE (CASE #2)

```
1
17.5, 3.14
1
17.5, 0.25, 4.19
17.5, 1.8, 3.6
3.14
11.08
1.0, 50.0, 1.0
1, 20.0
Y, Y
N
```

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Fig. No. 13

DATA CODING FORM

DATA FOR THE ROLL MOTION OF A BALLASTED POLE (CASE #2)

```
1.0, 50.0, 1.0
1, 20.0
1.0
17.5
3.0
5.0
2
2.0, 22.0, -1.0, 16.0, 13.0, 1.0, LIGHT TOP
2.0, 2.0, -1.0, 384.0, 1.0, 1.0, HEAVY BOTTOM
-1
Y, Y, 0.0, 15.0
N
```

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Fig. No. 14

PERIOD	ANG FREQ	RAO	W-F PHASE	F-H PHASE	W-H PHASE	AMP SPEC
1.000	.628E 01	.000	50.115	-179.027	-128.912	.002
2.000	.314E 01	.001	11.136	-178.863	-167.727	.055
3.000	.209E 01	.062	4.703	-178.420	-173.716	.406
4.000	.157E 01	.701	2.596	-176.519	-173.923	1.597
5.000	.126E 01	3.246	1.646	-6.319	-4.673	4.223
6.000	.105E 01	1.381	1.138	-1.427	-.289	8.094
7.000	.898E 00	1.153	.833	-.745	.089	11.382
8.000	.785E 00	1.078	.637	-.480	.157	11.477
9.000	.698E 00	1.044	.502	-.342	.160	7.928
10.000	.628E 00	1.028	.407	-.259	.148	3.524
11.000	.571E 00	1.018	.336	-.204	.132	.933
12.000	.524E 00	1.012	.282	-.166	.116	.135
13.000	.483E 00	1.009	.240	-.138	.103	.010
14.000	.449E 00	1.006	.207	-.116	.091	.000
15.000	.419E 00	1.005	.180	-.100	.081	.000
16.000	.393E 00	1.004	.158	-.086	.072	.000
17.000	.370E 00	1.003	.140	-.076	.065	.000
18.000	.349E 00	1.002	.125	-.067	.058	.000
19.000	.331E 00	1.002	.112	-.060	.053	.000
20.000	.314E 00	1.001	.101	-.054	.048	.000
21.000	.299E 00	1.001	.092	-.048	.044	.000
22.000	.286E 00	1.001	.084	-.044	.040	.000
23.000	.273E 00	1.001	.077	-.040	.037	.000
24.000	.262E 00	1.001	.070	-.037	.034	.000
25.000	.251E 00	1.001	.065	-.034	.031	.000
26.000	.242E 00	1.000	.060	-.031	.029	.000
27.000	.233E 00	1.000	.056	-.029	.027	.000
28.000	.224E 00	1.000	.052	-.027	.025	.000
29.000	.217E 00	1.000	.048	-.025	.023	.000
30.000	.209E 00	1.000	.045	-.023	.022	.000
31.000	.203E 00	1.000	.042	-.022	.021	.000
32.000	.196E 00	1.000	.040	-.020	.019	.000
33.000	.190E 00	1.000	.037	-.019	.018	.000
34.000	.185E 00	1.000	.035	-.018	.017	.000
35.000	.180E 00	1.000	.033	-.017	.016	.000
36.000	.175E 00	1.000	.031	-.016	.015	.000
37.000	.170E 00	1.000	.030	-.015	.015	.000
38.000	.165E 00	1.000	.028	-.014	.014	.000
39.000	.161E 00	1.000	.027	-.014	.013	.000
40.000	.157E 00	1.000	.025	-.013	.013	.000
41.000	.153E 00	1.000	.024	-.012	.012	.000
42.000	.150E 00	1.000	.023	-.012	.011	.000
43.000	.146E 00	1.000	.022	-.011	.011	.000
44.000	.143E 00	1.000	.021	-.011	.010	.000
45.000	.140E 00	1.000	.020	-.010	.010	.000
46.000	.137E 00	1.000	.019	-.010	.009	.000
47.000	.134E 00	1.000	.018	-.009	.009	.000
48.000	.131E 00	1.000	.018	-.009	.009	.000
49.000	.128E 00	1.000	.017	-.009	.009	.000
50.000	.126E 00	1.000	.016	-.008	.008	.000

Fig. No. 15

Heave Response Amplitude Operator
"Telephone Pole"

RAR IS IN DEGREES/FOOT OF WAVE AMPLITUDE

PERIOD	ANG FREQ	RAR	W-T PHASE	T-R PHASE	W-R PHASE	AMP SPEC
1.000	.628E 01	2.373	94.644	-172.529	-77.886	.002
2.000	.314E 01	1.898	114.370	-171.541	-57.171	.055
3.000	.209E 01	2.241	145.846	-169.158	-23.312	.406
4.000	.157E 01	4.270	163.392	-162.214	1.178	1.597
5.000	.126E 01	14.685	170.591	-112.391	58.200	4.223
6.000	.105E 01	5.609	173.937	-19.060	154.877	8.094
7.000	.898E 00	2.622	175.732	-8.363	167.369	11.382
8.000	.785E 00	1.638	176.806	-5.052	171.753	11.477
9.000	.698E 00	1.155	177.500	-3.485	174.015	7.928
10.000	.628E 00	.871	177.978	-2.587	175.391	3.524
11.000	.571E 00	.685	178.329	-2.014	176.315	.933
12.000	.524E 00	.556	178.578	-1.620	176.958	.135
13.000	.483E 00	.461	178.773	-1.336	177.437	.010
14.000	.449E 00	.390	178.930	-1.124	177.806	.000
15.000	.419E 00	.334	179.054	-.960	178.094	.000
16.000	.393E 00	.290	179.126	-.830	178.296	.000
17.000	.370E 00	.254	179.243	-.726	178.517	.000
18.000	.349E 00	.225	179.268	-.640	178.628	.000
19.000	.331E 00	.201	179.315	-.570	178.746	.000
20.000	.314E 00	.180	179.357	-.510	178.847	.000
21.000	.299E 00	.162	179.393	-.460	178.934	.000
22.000	.286E 00	.147	179.523	-.417	179.106	.000
23.000	.273E 00	.134	179.610	-.379	179.230	.000
24.000	.262E 00	.123	179.542	-.347	179.195	.000
25.000	.251E 00	.113	179.497	-.318	179.179	.000
26.000	.242E 00	.104	179.542	-.293	179.249	.000
27.000	.233E 00	.096	179.629	-.271	179.358	.000
28.000	.224E 00	.089	179.601	-.252	179.349	.000
29.000	.217E 00	.083	179.396	-.234	179.162	.000
30.000	.209E 00	.078	179.762	-.218	179.543	.000
31.000	.203E 00	.073	179.897	-.204	179.693	.000
32.000	.196E 00	.068	179.926	-.191	179.735	.000
33.000	.190E 00	.064	179.963	-.179	179.784	.000
34.000	.185E 00	.060	-179.955	-.169	-180.123	.000
35.000	.180E 00	.057	179.621	-.159	179.462	.000
36.000	.175E 00	.053	179.778	-.150	179.628	.000
37.000	.170E 00	.051	179.438	-.142	179.296	.000
38.000	.165E 00	.048	-179.805	-.134	-179.939	.000
39.000	.161E 00	.046	-179.875	-.128	-180.002	.000
40.000	.157E 00	.043	179.417	-.121	179.296	.000
41.000	.153E 00	.041	-179.352	-.115	-179.467	.000
42.000	.150E 00	.039	179.696	-.110	179.586	.000
43.000	.146E 00	.037	-179.358	-.105	-179.462	.000
44.000	.143E 00	.036	-179.559	-.100	-179.658	.000
45.000	.140E 00	.034	179.657	-.095	179.562	.000
46.000	.137E 00	.033	179.571	-.091	179.479	.000
47.000	.134E 00	.031	-178.071	-.087	-178.158	.000
48.000	.131E 00	.030	178.501	-.084	178.417	.000
49.000	.128E 00	.029	177.126	-.080	177.046	.000
50.000	.126E 00	.028	178.549	-.077	178.472	.000

Fig. No. 16

Roll Response Amplitude Operator
"Telephone Pole"
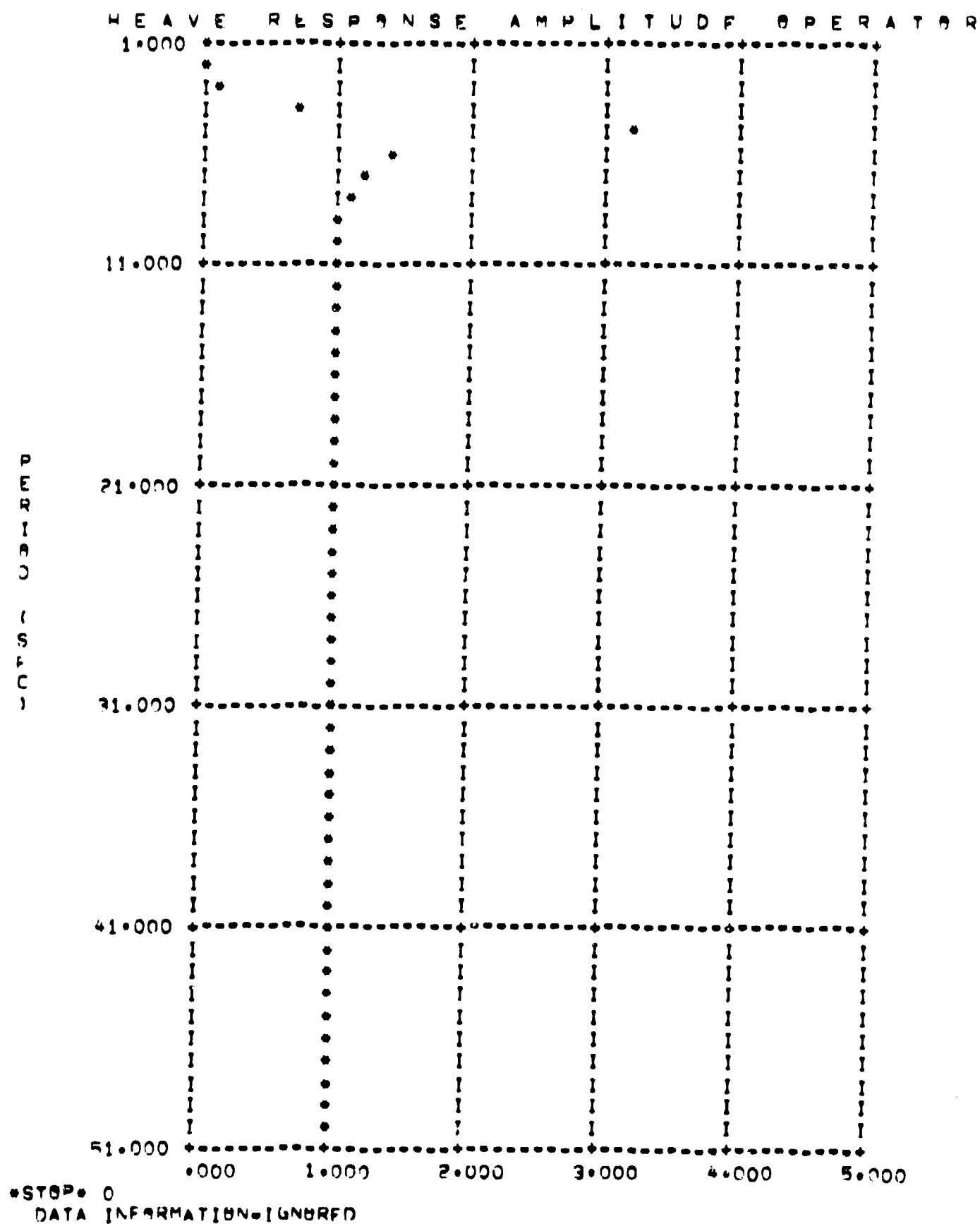
Fig. No. 17
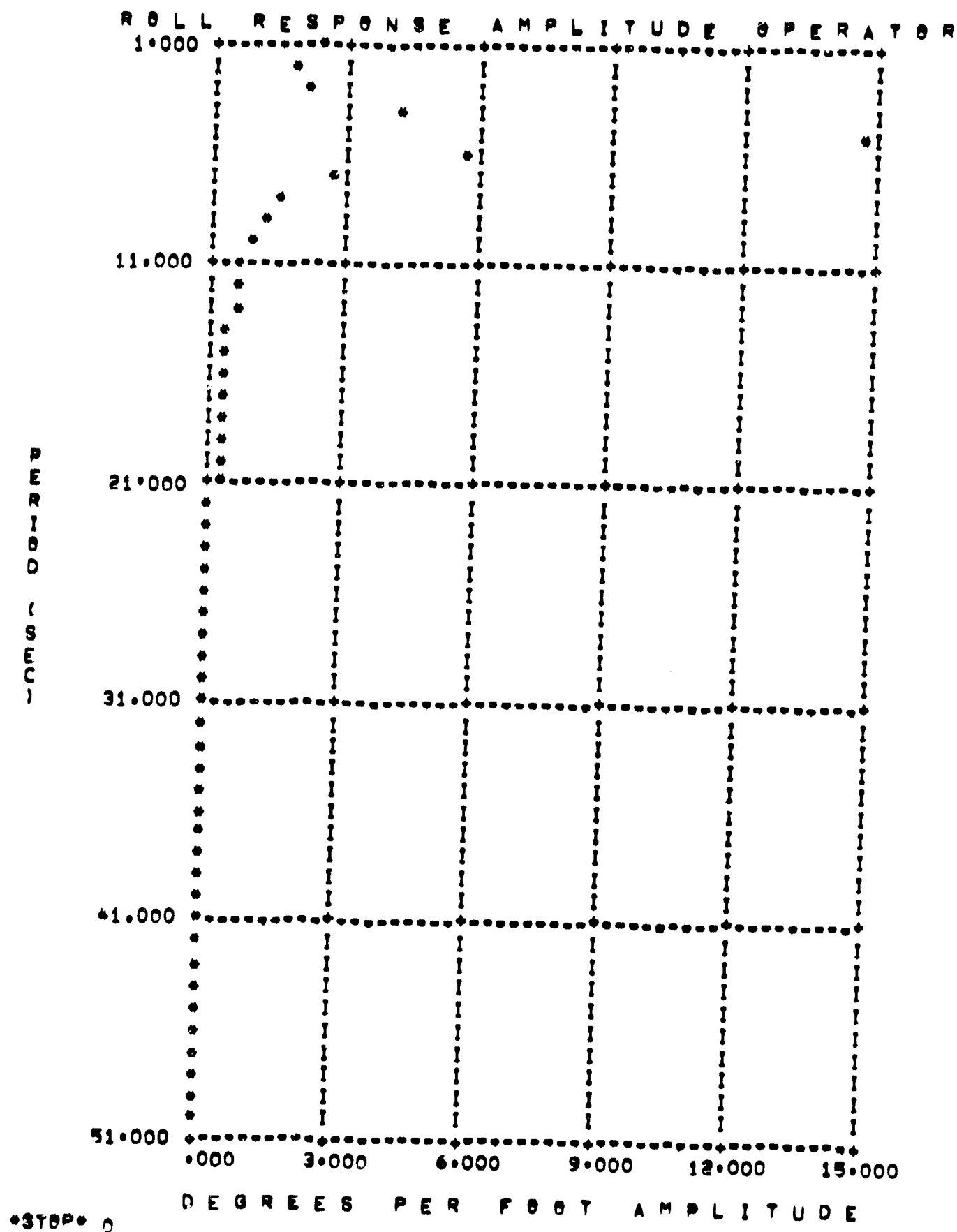
Fig. No. 18

RMS OF WAVE SPECTRUM =   2.683 FEET

PROBABLE AMPLITUDE
     OF WAVE   =   1.897 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE WAVE AMPLITUDE		NUMBER OF WAVES	EXPECTED WAVE MAXIMUM AMPLITUDE
----------	---------	I	--------	--------
.010	6.330	I	50	5.689
.100	4.830	I	100	6.118
.333	3.799	I	500	7.003
.500	3.370	I	1000	7.459
1.000	2.377	I	10000	8.399
		I	100000	9.311

RMS OF RESPONSE SPECTRUM =   4.362 FEET

PROBABLE AMPLITUDE
          OF HEAVE RESPONSE =   3.084 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE HEAVE AMPLITUDE RESPONSE		NUMBER OF WAVES	EXPECTED HEAVE MAXIMUM AMPLITUDE
----------	---------	I	--------	---------
.010	10.291	I	50	9.248
.100	7.852	I	100	9.946
.333	6.177	I	500	11.386
.500	5.479	I	1000	12.128
1.000	3.865	I	10000	13.654
		I	100000	15.130

Table No. 5   Heave Response of Telephone Pole

RMS OF WAVE SPECTRUM =   2.683 FEET

PROBABLE AMPLITUDE
    OF WAVE   =   1.897 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE WAVE AMPLITUDE		NUMBER OF WAVES	EXPECTED WAVE MAXIMUM AMPLITUDE
-----------	---------	I	--------	---------
.010	6.330	I	50	5.689
.100	4.830	I	100	6.118
.333	3.799	I	500	7.003
.500	3.370	I	1000	7.459
1.000	2.377	I	10000	8.399
		I	100000	9.311

RMS OF RESPONSE SPECTRUM =   17.684 DEG

PROBABLE AMPLITUDE
       OF ROLL RESPONSE   =   12.502 DEG

AMPLITUDES OF ROLL ARE IN DEGREES

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE ROLL AMPLITUDE RESPONSE		NUMBER OF WAVES	EXPECTED ROLL MAXIMUM AMPLITUDE
-----------	---------	I	--------	---------
.010	41.716	I	50	37.489
.100	31.831	I	100	40.319
.333	25.040	I	500	46.154
.500	22.211	I	1000	49.160
1.000	15.668	I	10000	55.350
		I	100000	61.362

Table No. 6   Roll Response of Telephone Pole
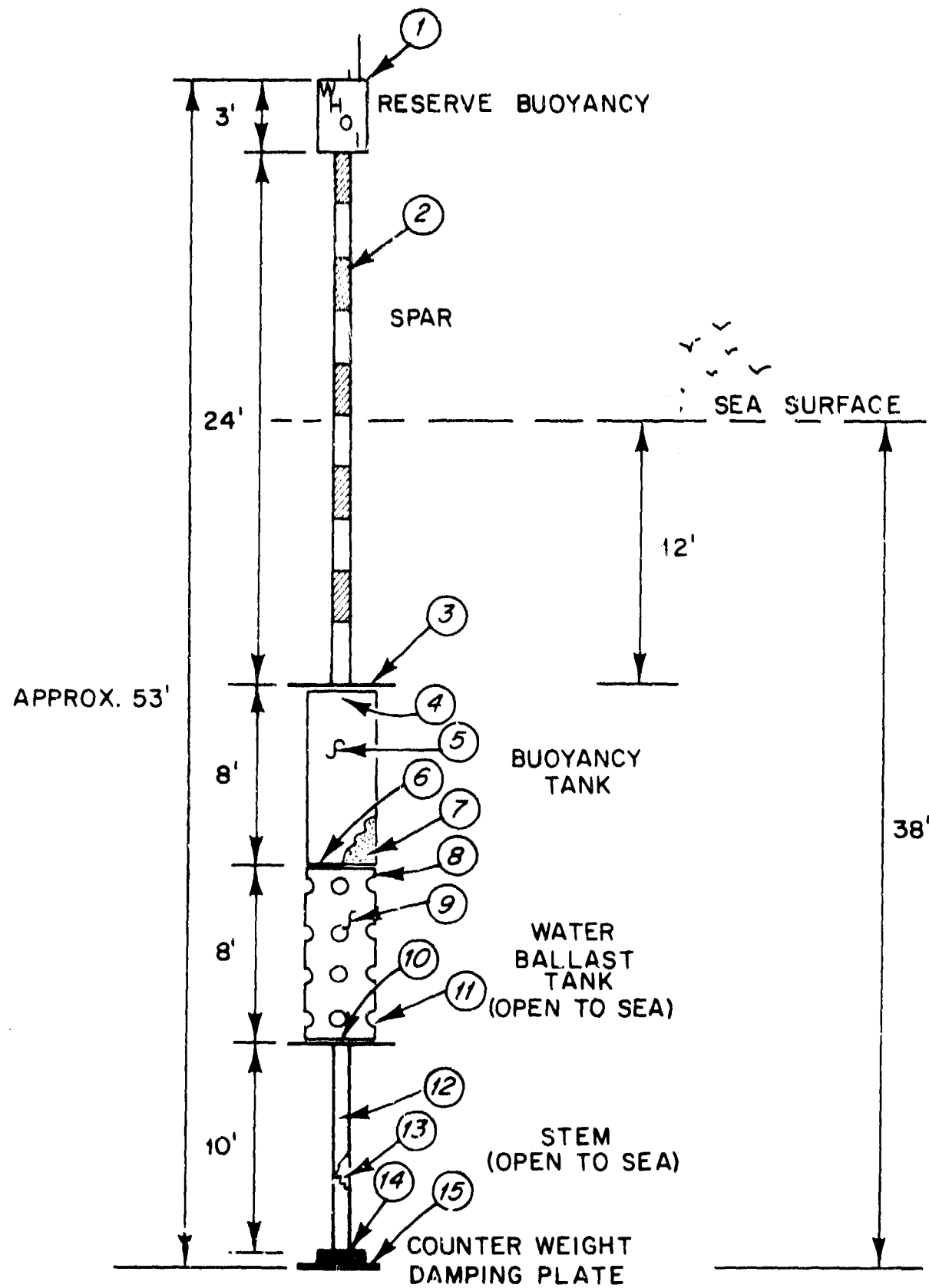
Fig. No. 19

Pierson-Moskowitz spectrum with a wind of 20 knots.

This relatively complex shape buoy is made of the following parts:

- A reserve buoyancy cylinder 3 ft. high, 2.5 ft. in diameter, made of 2 lbs/cu. ft. polyurethane foam.

- A spar 24 ft. long made of 8" O. D by 1/4" thick wall aluminum tubing.

- A spar base plate, made of 4 ft. diameter by 1/2" thick aluminum plate.

- A 3'.0" diameter by 8'.0" long buoyancy tank made of 3/16" steel plates. The buoyancy tank is filled with 4 lbs/cu. ft. foam.

- A 3'.0" diameter by 8'.0" water ballast tank made of 1/8" steel plates. The tank is filled with sea water. The bottom plate is 4'.0" in diameter.

- A 10'.0" long stem made of 6 5/8" O. D. schedule 40 steel pipe filled with sea water.

- A 4'.0" diameter by 1/2" thick damping plate.

- A counterweight cylinder 2.5 ft. in diameter by 0.848 ft. high, made of cast iron.

The physical parameters of the buoy main components are summarized in Table No. 7.

Table No. 7

Table of Spar Buoy Components Parameters

Part No.	Name	Shape	Width (ft) (diameter)	Height (ft)	Thick (ft) (wall)	Density (lbs/ft^3)	Weight (lbs)	C.G. Above Keel
1	Reserve Buoyancy	Solid Cyl.	2.5	3.0		2.0	29.45	52.383
2	Spar	Hollow Cyl.	0.666	24.3	0.0208	160.0	167.11	38.889
3	Plate Al.	Solid Cyl.	4.0	0.416		160.0	83.64	26.909
4	Plate St.	Solid Cyl.	3.0	0.0156		490.0	54.03	26.889
5	Plate St.	Hollow Cyl.	3.0	8.0	0.0156	490.0	573.72	22.889
6	Plate St.	Solid Cyl.	3.0	0.0156		490.0	54.03	18.900
7	Foam	Solid Cyl.	3.0	8.0		4.0	226.19	22.889
8	Plate St.	Solid Cyl.	3.0	0.0104		490.0	36.02	1.8844
9	Plate St.	Hollow Cyl.	3.0	8.0	0.0104	490.0	384.22	14.8896
10	Plate St.	Solid Cyl.	4.0	0.0104		490.0	64.03	10.8948
11	Water	Solid Cyl.	3.0	8.0		64.0	64.03	14.8896
12	Stem	Hollow Cyl.	3.0	8.0	0.0233	64.0	3619.11	14.8896
13	Water	Hollow Cyl.	0.552	10.0		490.0	197.98	5.8896
14	Counter-weight	Solid Cyl.	0.552	10.0		64.0	153.16	5.8896
15	Damping Plate	Solid Cyl.	2.5	0.848		450.0	1875.00	0.4656
		Solid Cyl.	4.0	0.0416		490.0	256.15	0.0208
						Total =	7773.84	

4.3.1   Program input

Computations and considerations in the support of pro-
gram input are as follows.

Buoy draft.   The weight of the water displaced by the
buoy equals the weight of the buoy.

Weight of water displaced by the stem

$$\frac{\pi}{4} (0.552)^2 \times 10 \times 64 \qquad = \qquad 153.16$$

Weight of water displaced by the ballast tank

$$\frac{\pi}{4} \times 3 \times 3 \times 8 \times 64 \qquad = \qquad 3619.11$$

Weight of water displaced by the buoyancy tank

$$\frac{\pi}{4} \times 3 \times 3 \times 8 \times 64 \qquad = \qquad 3619.11$$

Weight of water displaced by the immersed portion of
the spar of length "h"

$$\frac{\pi}{4} \times (0.666)^2 h \times 64 = 22.295 h$$

Solving for h

$$h = \frac{7773.84 - (153.16 + 2 \times 3.619.11)}{22.295} \cong 12.0'$$

The buoy draft is therefore 38'.

Number, depth and area of pressure surfaces.   There
are two pressure surfaces to consider, namely the top and the
bottom of the foam filled buoyancy tank.   The area of the first
pressure surface S_1 is given by

$$S_1 = \pi \left(R_1^2 - R_2^2 \right) \qquad \text{where}$$

R_1 is the radius of the tank, $R_1 = 1.5'$

R_2 is the radius of the spar, $R_2 = 0.33'$

Thus $S_1 = \pi(1.5^2 - 0.33^2) = 6.71$ sq. ft.

S_1 is located 12' below the surface.   The pressure force acting
on S_1 being downwards, S_1 is negative.   The area of the second

pressure surface S_2 is in turn given by

$$S_2 = \pi R_1^{\,2} = 7.06 \text{ sq. ft.}$$

It is located 20' below the surface. The pressure force is upwards and thus S_2 is positive.

Number, depth, added mass coefficients, and volume of inertial components. For computing the heave response, two distinct added mass effects must be accounted for: the added mass due to the water entrained by the top and bottom plates of the buoyancy and water ballast tanks, and the added mass due to the water entrained by the damping plate.

The first added mass effect will be assumed to be the same as the one produced by a sphere with a diameter equal to the diameter (4') of the plates located at the top of the buoyancy tank and the bottom of the water ballast tank. The equivalent depth will be selected midway between the two plates, i.e. 20' below the surface. The added mass coefficient for a sphere is $\frac{1}{2}$. The volume VOL_1 of this first inertial component is

$$VOL_1 = \frac{4}{3} \pi \, (2)^3 = 33.51 \text{ cu. ft.}$$

The second added mass effect will be considered to take place at the buoy keel, i.e. at 38' below the surface. The formula for the added mass of a circular plate of radius "a" being

$$m' = \frac{8}{3} \rho \, a^3 = C_m \rho \, VOL$$

An arbitrary added mass coefficient of 1 will yield a volume VOL_2

$$VOL_2 = \frac{8}{3} a^3 = \frac{8}{3}(2)^3 = 21.33 \ cu.ft$$

Both effects in this case are happening all the time.

Number, depth, damping and wave drag coefficients of drag surfaces. Damping and wave drag will be assumed to be produced mainly by or on the upper and lower faces of the buoyancy tank and water ballast tank and by the damping plate at the buoy lower end. There will thus be two drag surfaces, one assumed to be located half way between the two ends of the tanks at an equivalent depth of 20', and the other at the buoy keel 38' below the surface. Assuming $\overline{X}_b = \frac{1}{2} \overline{X}_c$ and $\overline{X}_c$ = 3' the corresponding damping and wave drag coefficients given by expressions (2.2.10) and (2.2.13) are found to be

$$b_1' = \frac{4}{3\pi} \times 2 \times 0.9 \times \pi (2)^2 \times 1.5 \ = \ 14.40$$

$$b_2' = \frac{4}{3\pi} \times 2 \times 1.2 \times \pi (2)^2 \times 1.5 \ = \ 19.20$$

$$c_1' = \frac{4}{3\pi} \times 2 \times 0.9 \times \pi (2)^2 \times 3 \ = \ 28.80$$

$$c_2' = \frac{4}{3\pi} \times 2 \times 1.2 \times \pi (2)^2 \times 3 \ = \ 38.40$$

Cross sectional area at surface.

$$\pi R^2 = \pi \times 0.33^2 = 0.342 \ sq.ft$$

Buoy virtual mass.

Added mass of first inertial component

$$m_1' = \frac{1}{2} \times \frac{64}{32.2} \times \frac{4}{3\pi} \times (2)^3 \qquad = \quad 33.3 \quad \text{slugs}$$

added mass of second inertial component

$$m_2' = \frac{8}{3} \times \frac{64}{32.2} \times (2)^3 \qquad = \quad 42.4 \quad \text{slugs}$$

Buoy mass "m"

$$m = \frac{7773.84}{32.2} \qquad = \quad \underline{237.85} \text{ slugs}$$

Virtual mass $m_v = m + m_1' + m_2' \qquad = \quad 313.55$ slugs

As in the preceding case study, the program user must also provide an arbitrary value of average buoy roll $\overline{\Theta}$ and wave amplitude $\overline{A}$. In this case $\overline{\Theta}$ and $\overline{A}$ are selected to be 5° and 2.5 feet respectively.

All pertinent data are listed in the data coding form shown in Fig. Nos. 20 and 21.

4.3.2 Program output

The heave and roll response amplitude operators are graphically represented in Fig. No. 22 and Fig. No. 23. The expected average and maximum values of wave amplitudes and of heave and roll motion are summarized in Table Nos. 8 and 9.

As a point of interest, Table No. 10 presents a succinct performance comparison of the three buoy types when submitted to the same random excitation.

DATA CODING FORM

DATA FOR THE HEAVE MOTION OF A WHOI BUOY (CASE #3)

```
2
12.0, -6.71
20.0, 7.06
2
20.0, 0.5, 33.51
38.0, 1.0, 21.33
2
20.0, 14.4, 28.8
38.0, 19.2, 38.4
0.342
313.55
1.0, 50.0, 1.0
1, 20.0
Y,Y
N
```

- 66a -

PROGRAM NUMBER   DATE   PAGE   OF

Fig. No. 20

INFORMATION PROCESSING CENTER
WOODS HOLE OCEANOGRAPHIC INSTITUTION
WOODS HOLE, MASSACHUSETTS

DATA CODING FORM

DATA FOR THE ROLL MOTION OF THE WHOI BUOY (CASE #3)

1.0, 50.0, 1.0
1, 20, 0
0.33
38.0
2.5
5.0
15

- 66b -

2	2.500	3.0	-1.0	21.0	152.389	-1.0 RESERVE BUOYAN
1	0.666	24.0	0.250	160.0	38.889	1.0 SPAR
2	4.000	0.0416	-1.0	160.0	26.309	1.0 PLATE AL.
2	3.000	0.0156	-1.0	490.0	26.889	1.0 PLATE ST.
1	3.000	8.0	0.187	490.0	27.889	1.0 PLATE ST.
2	3.000	0.0156	-1.0	490.0	18.900	1.0 PLATE ST.
2	3.000	8.0	-1.0	4.0	22.889	1.0 FOAM
2	3.000	0.0104	-1.0	490.0	18.884	1.0 PLATE ST.
1	3.000	8.0	0.125	490.0	14.889	1.0 PLATE ST.
2	4.000	0.0104	-1.0	490.0	10.895	1.0 PLATE ST.
2	3.000	8.0	-1.0	64.0	14.889	1.0 WATER
1	0.552	10.0	0.280	490.0	5.889	1.0 STEM
2	0.552	10.0	-1.0	64.0	5.889	1.0 WATER
2	2.500	0.848	-1.0	450.0	0.466	1.0 COUNTERWEIGHT
2	4.000	0.0416	-1.0	490.0	0.021	1.0 DAMPING PLATE

-1
Y,Y
N

Fig. No. 21
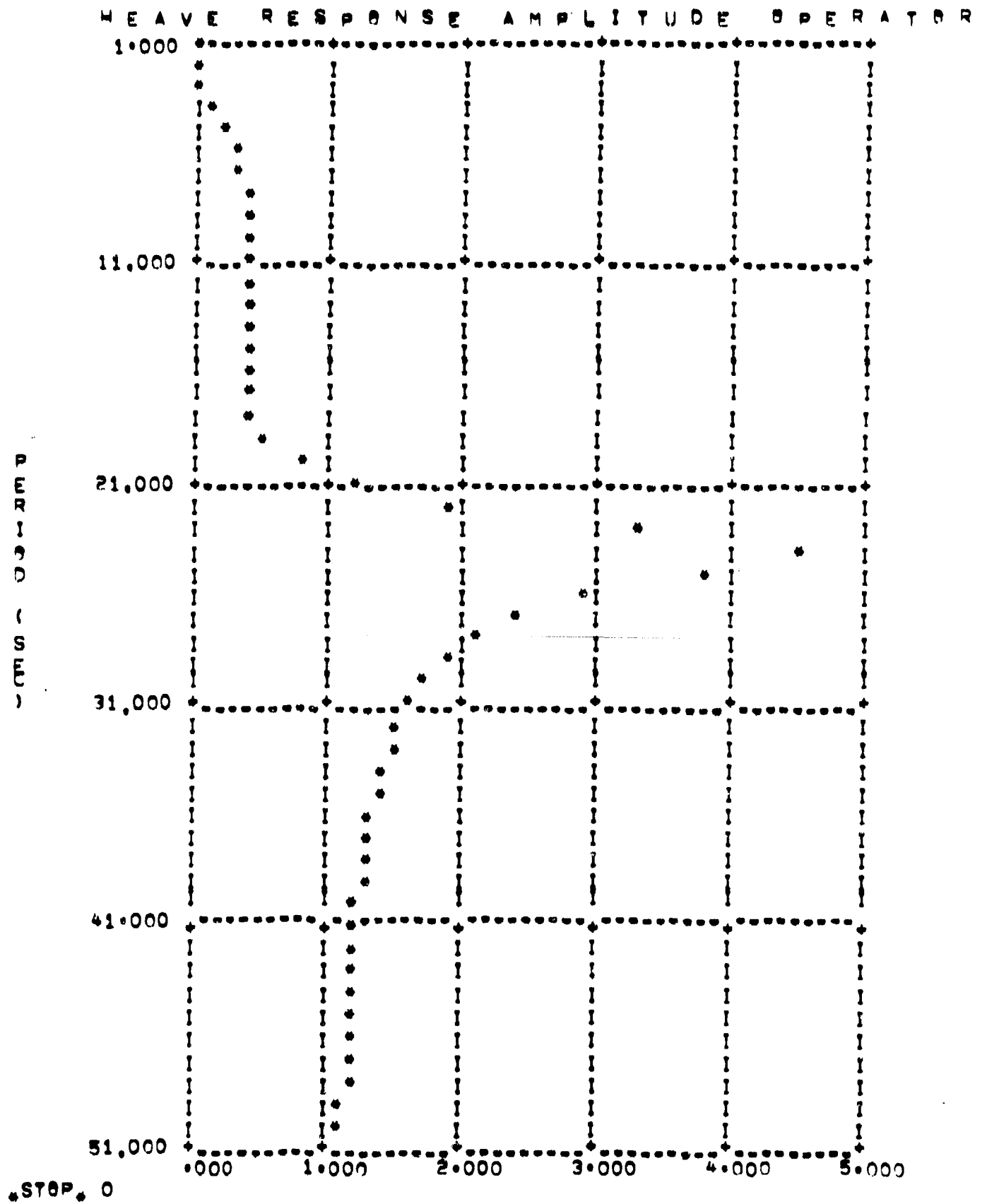
Fig. No. 22
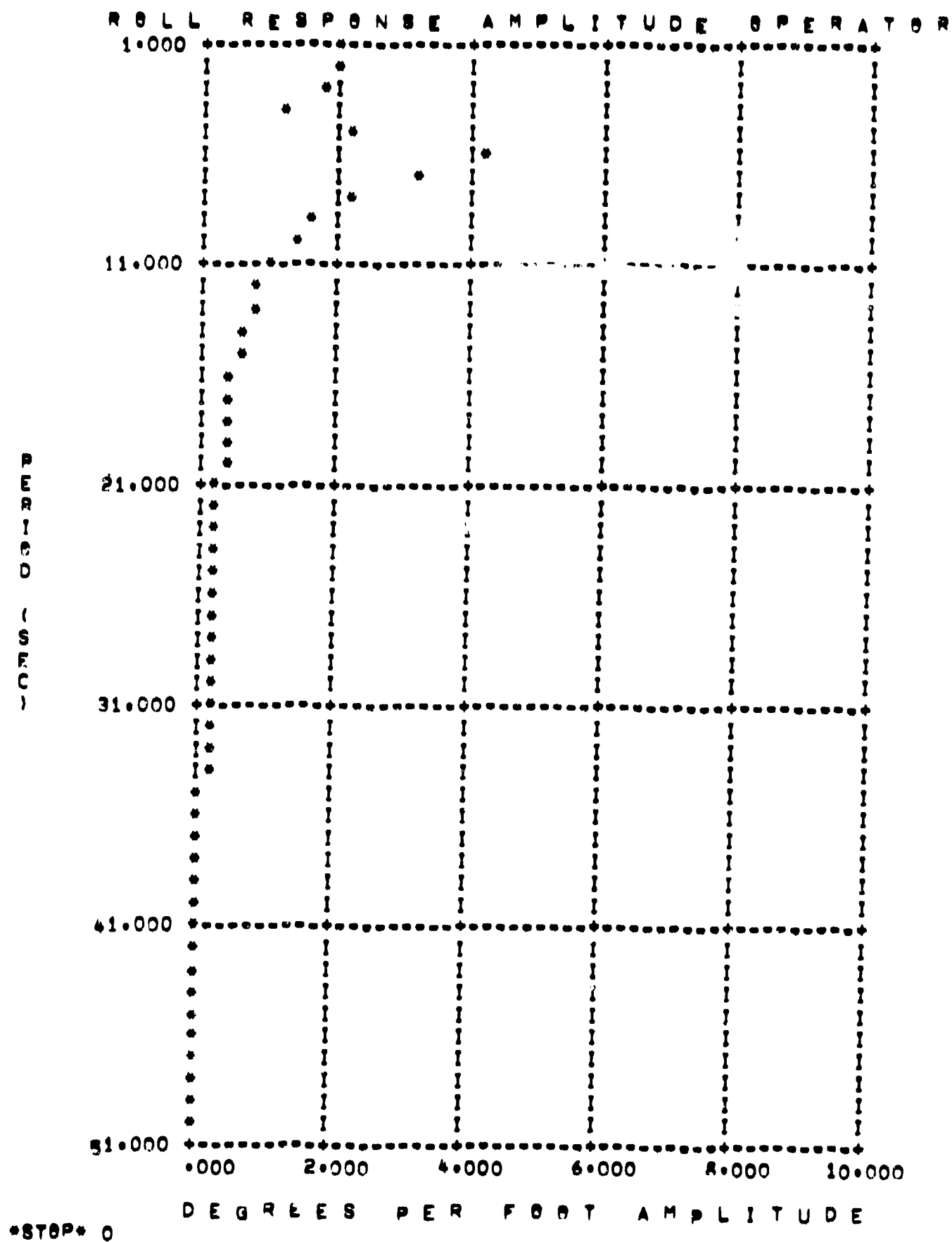
Fig. No. 23

RMS OF WAVE SPECTRUM = 2.683 FEET

PROBABLE AMPLITUDE
OF WAVE = 1.897 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE WAVE AMPLITUDE	NUMBER OF WAVES	EXPECTED WAVE MAXIMUM AMPLITUDE
.010	6.330	50	5.689
.100	4.830	100	6.118
.333	3.799	500	7.003
.500	3.370	1000	7.459
1.000	2.377	10000	8.399
		100000	9.311

RMS OF RESPONSE SPECTRUM = .809 FEET

PROBABLE AMPLITUDE
OF HEAVE RESPONSE = .572 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE HEAVE AMPLITUDE RESPONSE	NUMBER OF WAVES	EXPECTED HEAVE MAXIMUM AMPLITUDE
.010	1.908	50	1.714
.100	1.456	100	1.844
.333	1.145	500	2.111
.500	1.016	1000	2.248
1.000	.716	10000	2.531
		100000	2.806

Table No. 8   Heave Response of Complex Shape Buoy

RMS OF WAVE SPECTRUM =   2.683 FEET

PROBABLE AMPLITUDE
   OF WAVE   =   1.897 FEET

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE WAVE AMPLITUDE	NUMBER OF WAVES	EXPECTED WAVE MAXIMUM AMPLITUDE
.010	6.330	50	5.689
.100	4.830	100	6.118
.333	3.799	500	7.003
.500	3.370	1000	7.459
1.000	2.377	10000	8.399
		100000	9.311

RMS OF RESPONSE SPECTRUM =   7.489 DEG

PROBABLE AMPLITUDE
      OF ROLL RESPONSE =   5.295 DEG

AMPLITUDES OF ROLL ARE IN DEGREES

FRACTION OF LARGEST AMPLITUDES CONSIDERED	AVERAGE ROLL AMPLITUDE RESPONSE	NUMBER OF WAVES	EXPECTED ROLL MAXIMUM AMPLITUDE
.010	17.667	50	15.877
.100	13.481	100	17.076
.333	10.605	500	19.547
.500	9.407	1000	20.820
1.000	6.636	10000	23.442
		100000	25.988

Table No. 9   Roll Response of Complex Shape Buoy

Table No. 10

Performance Comparison

Buoy Type	Average Heave / Average Wave Amplitude	Significant Heave / Average Wave Amplitude
Flat Cylinder	1.000	3.961
Telephone Pole	1.625	6.591
Spar Buoy	0.301	2.791

5.0   CONCLUSIONS AND LIMITATIONS

The theoretical introduction and the case studies presented point
out the positive aspects of the computer solution as well as some of its
limitations.   In the formulation of the equations of heave and roll motion,
an attempt has been made to account for the effects of the water particle
velocity and acceleration.   The depth dependence of these effects has
been included.   The model thus obtained is more realistic than simpler
models which consider only buoy displacement and wave slope as the
predominant exciting forces.

As illustrated in case study number 3, the response of buoys of
relatively complex shape can be easily studied.   The heave and roll
response amplitude operators can be used to compute the response of
the buoys to waves of known or specified amplitude and frequencies.   In
addition to this time domain approach, specified spectral densities can
be used to derive certain statistical expectancies of buoy heave and roll
amplitudes.   Parametric studies of buoy performance can thus readily
be made.

On the other hand, to satisfy the condition of linearity, certain
assumptions are made which introduce in the solution a degree of
arbitrariness difficult to evaluate.   Certainly the initial choice of the
average values of wave amplitude, buoy heave, and buoy roll angle
used to compute the linearized coefficients of drag and inertia will
reflect on the accuracy of the solution.   To improve this accuracy an
iterative procedure can be followed which replaces the initial assumed

values by computed ones until sufficient agreement is achieved.

Experimentally verified values of linear equivalents of inertia and viscous effects would greatly help validate or improve the computer solution described in this report. The assumption of small roll angles, also required by the condition of linearity, further limits the use of this program.

Energy dissipation by wave radiation is not considered. This factor could be important in large disk buoys. Finally the effects that mooring lines and tether lines would have on the buoy response have not been included, thus restricting this solution to free floating buoys.

Despite these limitations, the rationale used in the derivation of the solution and the program input flexibility make the computer solution useful as well as practical.

6.0 REFERENCES

1. Berteaux, H. O., Buoy Engineering, John Wiley & Sons, Inc., 1976.

2. Marks, W., "The Application of Spectral Analysis and Statistics to Seakeeping", The Society of Naval Architects and Marine Engineers , No. 1-24, September, 1963.

3. Price, W. G. and R.E.D. Bishop, Probabilistic Theory of Ship Dynamics, Chapman and Hall, 1974.

7.0 APPENDICES

<div align="center">APPENDIX I</div>

Expression of Linearized Damping Coefficient

When the drag force on a body moving with a velocity V is assumed to be linearly proportional to the velocity, the expression of the force is simply

$$D_1 = \alpha V$$

In most cases, however, drag forces are expressed in terms of V^2, using the familiar formula

$$D_2 = \frac{1}{2} \rho \, C_D \, A \, V^2$$

where ρ = water mass density

C_D = drag coefficient

A = body area across the flow (blunt bodies).

If the motion of the body is periodic, with amplitude X_o' and frequency $\omega = \dfrac{2\pi}{T}$, i.e. if for example

$$X = X_o' \sin \omega t$$

then the amount of energy dissipated per cycle by the damping force is given by

$$E = \int_0^T D_2 \, d\ell = \int_0^T D_2 \, V dt = \frac{1}{2} \rho \, C_D \, A \int_0^T \omega^3 X_o'^3 (\cos \omega t)^3 \, dt$$

or

$$E = \frac{4}{3} \rho \, C_D \, A \, X_o'^3 \, \omega^2$$

The amount of energy dissipated per cycle by the linear damping force is in turn given by

$$E = \int_0^T D_1 \, d\ell = \int_0^T D_1 \, V dt = \alpha \int_0^T \omega^2 X_o'^2 (\cos \omega t)^2 \, dt$$

or

$$E = \pi \alpha \, X_o'^2 \, \omega$$

Assuming both forces to dissipate the same amount of energy will yield the expression of the linearized damping coefficient d , namely

$$d = \frac{4}{3\pi} \, \rho \, C_D \, A X_o \, \omega$$

APPENDIX II

Evaluation of the Coefficient "B" of Damping Moment

"B" has been previously defined as:

$$B = \alpha\omega \left\{ \int_{r_1=0}^{r_1=S} d(r_1)\, r_1^3\, dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)\, r_2^3\, dr_2 \right\}$$
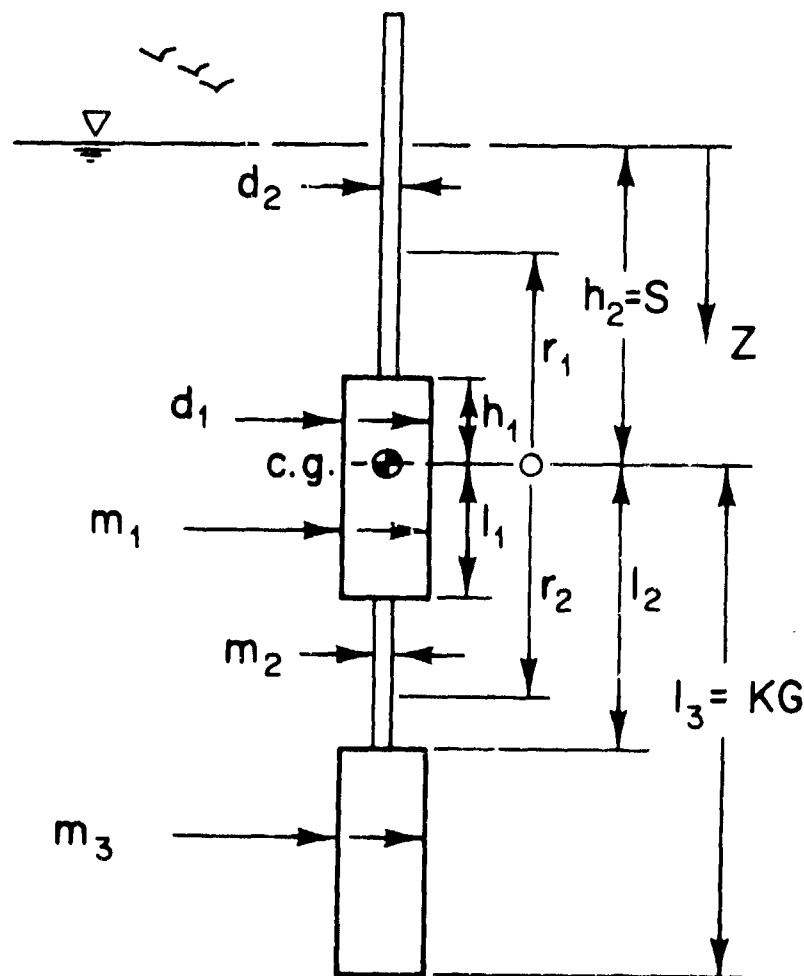
Fig. No. 24

Let us consider a buoy made of different cylinders as shown on the sketch. (Fig. No. 24) Let d_1, d_2, be the values of $d(r_1)$ and h_1 and h_2 be the corresponding limits of the variable r_1. The integration of the first integral

$$\int_{r_1=0}^{r_1=S} r_1^3\, d(r_1)\, dr_1$$

yields:

$$d_1 \int_{r_1=0}^{r_1=h_1} r_1^3 dr_1 + d_2 \int_{r_1=h_1}^{r_1=h_2} r_1^3 dr_1 = \frac{d_1 h_2^4}{4} + \frac{d_2 h_2^4}{4} - \frac{d_2 h_1^4}{4}$$

This result obviously leads to the recurrence formula

$$\int_{r_1=0}^{r_1=S} r_1^3 d(r_1) dr_1 = \frac{1}{4} \sum_i h_i^4 (d_i - d_{i+1}) \qquad i = 1,2$$

Let now m_1, m_2, m_3 be the values of $d(r_2)$ and ℓ_1, ℓ_2, ℓ_3 the corresponding limits of the variable r_2. The second integral can then be evaluated as follows:

$$\int_{r_2=0}^{r_2=\overline{AG}} r_2^3 d(r_2) dr_2 = m_1 \int_{r_2=0}^{r_2=\ell_1} r_2^3 dr_2 + m_2 \int_{r_2=\ell_1}^{r_2=\ell_2} r_2^3 dr_2 + m_3 \int_{r_2=\ell_2}^{r_2=\ell_3} r_2^3 dr_2$$

$$= \frac{1}{4} \left\{ \ell_1^4 (m_1 - m_2) + \ell_2^4 (m_2 - m_3) + \ell_3^4 m_3 \right\}$$

This result in turn yields to the recurrence formula

$$\int_{r_2=0}^{r_2=\overline{AG}} d(r_2) r_2^3 dr_2 = \frac{1}{4} \sum_j \ell_j^4 (m_j - m_{j+1}) \qquad j = 1,2,3$$

The expression of the coefficient "B" is therefore

$$B = \frac{\alpha \omega}{4} \left\{ \sum_i h_i^4 (d_i - d_{i+1}) + \sum_j l_j^4 (m_j - m_{j+1}) \right\}$$

where

$$V = \frac{4}{3\pi} \rho \, C_D \, \bar{\theta}$$

APPENDIX III

Evaluation of the Coefficient "D" of Wave Drag Moment

The coefficient "D" has been previously defined as

$$D = \beta \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1 e^{-2Kz} dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2 e^{-2Kz} dr_2 \right\}$$

The integrals can be readily evaluated with the help of the following argument. For small angles Θ, the projection of μ on the vertical is approximately equal to μ.

Thus, from $r_1 = 0$ to $r_1 = S$, $z \cong S - r_1$

and from $r_2 = 0$ to $r_2 = \overline{KG}$, $z \cong S + r_2$

Introducing these values of z in the integrals yields

$$D = \beta e^{-2KS} \left\{ \int_{r_1=0}^{r_1=S} d(r_1) e^{2Kr_1} r_1 dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2 e^{-2Kr_2} dr_2 \right\}$$

Considering the same buoy geometry as in Appendix II, and evaluating the first integral over the domain of variation of r_1 yield:

$$\frac{1}{4K^2} \left\{ d_1 \left[e^{2Kh_1}(2Kh_1 - 1) + 1 \right] + d_2 \left[e^{2Kh_2}(2Kh_2 - 1) - e^{2Kh_1}(2Kh_1 - 1) \right] \right\}$$

This result leads to the recurrence formula:

$$\int_{r_1=0}^{r_1=S} d(r_1) r_1 e^{2Kr_1} dr_1 = \frac{1}{4K^2} \sum d_i \left[e^{2Kh_i}(2Kh_i - 1) - e^{2Kh_{i-1}}(2Kh_{i-1} - 1) \right]$$

$$i = 1, 2$$

Similarly, the evaluation of the second integral over the domain of variation

of r_2' yields:

$$
\frac{m_1}{4K^2}\left[e^{-2K\ell_1}(2K\ell_1+1)-1\right]
$$
$$
+\frac{m_2}{4K^2}\left[e^{-2K\ell_2}(2K\ell_2+1)-e^{-2K\ell_1}(2K\ell_1+1)\right]
$$
$$
+\frac{m_3}{4K^2}\left[e^{-2K\ell_3}(2K\ell_3+1)-e^{-2K\ell_2}(2K\ell_2+1)\right]
$$

The recurrence formula thus is

$$
-\int_{r_2=0}^{r_2'=\overline{Ka}}d(r_2)\,r_2'\,e^{-2Kr_2}\,dr_2=\frac{1}{4K^2}\sum_j m_j\left[e^{-2K\ell_j'}(2K\ell_j'+1)-e^{-2K\ell_{j-1}'}(2K\ell_{j-1}'+1)\right]_{j=1,2,3}
$$

Thus, the expression of the coefficient "D" is finally

$$
D=\frac{\beta e^{-2Ks}}{4K^2}\left\{\sum_i d_i\left[e^{2Kh_i}(2Kh_i-1)-e^{2Kh_{i-1}}(2Kh_{i-1}-1)\right]\right.
$$
$$
\left.+\sum_j m_j\left[e^{-2K\ell_j'}(2K\ell_j'+1)-e^{-2K\ell_{j-1}'}(2K\ell_{j-1}'+1)\right]\right\}
$$

with $\quad K=\dfrac{\omega^2}{g}$

and $\quad \beta=\dfrac{4}{3\pi}\rho\,C_D\,\overline{A_F}\,\omega$

APPENDIX IV

Evaluation of the Coefficient "P" of Wave Inertia Moment

"P" has been previously defined as

$$P = r \left\{ -\int_{r_1=0}^{r_1=s} d(r_1)^2 r_1 e^{-Kz} dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)^2 r_2 e^{-Kz} dr_2 \right\}$$

Using the change of variables previously discussed, the integrals in the bracket can be written:

$$-\int_0^s d(r_1)^2 r_1 e^{-K(s-r_1)} dr_1 + \int_0^{\overline{KG}} d(r_2)^2 r_2 e^{-K(s+r_2)} dr_2$$

or

$$e^{-Ks} \left\{ -\int_0^s d(r_1)^2 r_1 e^{Kr_1} dr_1 + \int_0^{\overline{KG}} d(r_2)^2 r_2 e^{-Kr_2} dr_2 \right\}$$

Noting that

$$-\int_a^b r_1 e^{Kr_1} dr_1 = \left. \frac{e^{Kr_1}}{K^2} (Kr_1 - 1) \right|_b^a$$

Then, over the intervals

$$0 \le r_1 \le h_1 \quad \text{with} \quad d(r_1) = d_1$$
$$h_1 \le r_1 \le h_2 \qquad d(r_2) = d_2$$

the evaluation of the integrals yield:

1) over the first interval

$$-\frac{d_1^2}{K^2} \left[1 + e^{Kh_1}(Kh_1 - 1) \right]$$

2) over the second interval,

$$\frac{d_2^2}{K^2}\left[e^{-Kh_1}(Kh_1-1) - e^{Kh_2}(Kh_2-1)\right]$$

It thus appears that

$$-\int_{r_1=0}^{r_1=S} d(r_1)^2 r_1 e^{-Kz} dr_1 = \frac{e^{-KS}}{K^2}\sum_i d_i^2\left[e^{-Kh_{i-1}}(Kh_{i-1}-1) - e^{Kh_i}(Kh_i-1)\right]$$

$$i = 1, 2$$

As far as the second integral is concerned, noting

$$\int r_2 e^{-Kr_2} dr_2 = \frac{e^{-Kr_2}}{K^2}(-Kr_2-1)\Big|_a^b$$

$$= \frac{e^{-Kr_2}}{K^2}(Kr_2+1)\Big|_b^a$$

and evaluating over the range $\ell_1 \leq r_2 \leq \ell_2$, $(d(r_2) = m_2)$

yield:

$$\frac{m_2^2}{K^2}\left\{e^{-K\ell_1}(K\ell_1+1) - e^{-K\ell_2}(K\ell_2+1)\right\}$$

which shows that

$$\int_0^{\overline{K_G}} d(r_2)^2 r_2 e^{-Kz} dr_2 = \frac{e^{-KS}}{K^2}\sum_j m_j^2\left[e^{-K\ell_{j-1}}(K\ell_{j-1}+1) - e^{-K\ell_j}(K\ell_j+1)\right]$$

$$j = 1, 2, 3$$

The expression of the coefficient "P" is therefore

$$P = \frac{\gamma\, e^{-KS}}{K^2} \left\{ \sum_i d_i^2 \left[e^{-Kh_{i-1}}(Kh_{i-1}-1) - e^{Kh_i}(Kh_i-1) \right] \right.$$
$$\left. + \sum_j m_j^2 \left[e^{-K\ell_{j-1}}(K\ell_{j-1}+1) - e^{-K\ell_j}(K\ell_j+1) \right] \right\}$$

with

$$K = \omega^2/g$$

$$\gamma = \frac{\pi C_m}{4}\,\rho$$

APPENDIX V

Evaluation of the Coefficient "I_{ρ}" of Added Moment of Inertia

The moment of inertia of the water displaced by the buoy with respect to the buoy c.g. is evaluated with the help of the parallel axis theorem, and is given by

$$\bar{I}_F = \sum_i I(\bar{x})_i + \sum_i M_i \left(\bar{x}_i - \bar{KG}\right)^2$$

where $\quad i = 1, 2, 3, \ldots m$

$I(\bar{x})_i$ = moment of inertia of cylinder "i" with respect to its own c.g.

$$I(\bar{x})_i = \frac{M_i}{4} \left(R_i^2 + \frac{H_i^2}{3}\right)$$

with M_i = mass of water displaced by cylinder "i"

R_i = radius of cylinder "i"

H_i = height of cylinder "i"

$\bar{X}_i$ = distance of c.g. of cylinder "i" to keel

$\bar{KG}$ = distance of buoy c.g. to keel.

APPENDIX VI

Computation Method for Coefficients "B", "D" and "P"

The actual computation of the roll response amplitude operator in computer program ROLLRAO is performed using different forms of the expressions for some of the moments. This was done because the buoy configuration is input as geometrical "solids" rather than surfaces. It was, therefore, more straightforward to implement the computation of the damping forces using an iterative procedure on the components. The following expressions give the form of the equations used.

For the buoy damping moment, "B"

$$B = \frac{\alpha \omega}{4} \sum_{i=1}^{i=n.} d_i \, sign(z - z_{cg}) \times (z - z_{cg})^4 \Big|_{z=z_{Ti}}^{z=z_{Bi}}$$

where $\alpha = \dfrac{4 \rho C_D \bar{\theta}}{3 \pi}$ as defined in Appendix II

d_i = diameter of the ith buoy component

z_{cg} = depth to the buoy center of gravity

z_{Bi} = depth to the bottom surface of the ith buoy component

z_{Ti} = depth to the top surface of the ith buoy component

$$sign(z - z_{cg}) = \frac{z - z_{cg}}{|z - z_{cg}|}$$

n = number of buoy components

For the water damping moment, "D"

$$D = \frac{\beta e^{-2Kz_{cg}}}{4K^2} \sum_{i=1}^{i=n} d_i \left[\left(-2K(z - z_{cg}) - 1.0 \right) e^{-2K(z - z_{cg})} + 1.0 \right] \Big|_{z=z_{Ti}}^{z=z_{Bi}}$$

where $\qquad \beta = \dfrac{\omega \, 4\rho \, \bar{A} \, C_D}{3\pi}$

and $K = \omega^2 / g$ as defined in Appendix III.

For the water particle acceleration inertia moment "P"

$$P = -\frac{\gamma e^{-Kz_{cg}}}{K^2} \sum_{i=1}^{i=m} d_i{}^2 \left[\left(-K(z-z_{cg}) - 1.0\right) e^{-K(z-z_{cg})} + 1.0 \right]\Bigg|_{z=z_{T_i}}^{z=z_{B_i}}$$

where $\gamma = \dfrac{\pi \rho C_m}{4}$ as defined in Appendix IV.

APPENDIX VII

Heave Program Listing

```
1.   C                        PROGRAM HERA0
2.   C
3.   C           VERSION 2.0    JAN, 1977        R. GOLDSMITH
4.   C           VERSION 1.1    JUNE, 1976       R. GOLDSMITH
5.   C
6.   C           THIS PROGRAM IS USED TO COMPUTE THE HEAVE RESPONSE
7.   C              AMPLITUDE OPERATOR, AND ASSOCIATED PHASE ANGLES, FOR
8.   C              SPAR TYPE BUOY SYSTEMS
9.   C
10.  C           VERSION 2.0 - MODIFIED TO INCORPORATE A WAVE DRAG COEFF
11.  C
12.  C
13.  C        LOGICAL IAMTERM
14.  C
15.          DIMENSION DEPTHP(25),AREA(25)
16.          DIMENSION DEPTHI(25),ADDMSC(25),VOLUME(25)
17.          DIMENSION DEPTHD(25),DAMPC(25),WDRAGC(25)
18.          DIMENSION MAXWAVNO(6),WVMAXCOF(6),HEVMAXHT(6)
19.          DIMENSION FRACAMPS(5),AVRCOEFF(5),AVRESPNS(5)
20.  C
21.          DATA NCR,NLP/105,108/
22.          DATA NMAX/25/
23.          DATA PI/3.141592/,RTOD/57.2958/
24.          DATA RHO,G/1.99035, 32.174/
25.  C
26.          DATA FRACAMPS /0.01,0.10,0.333,0.50,1.0 /
27.          DATA AVRCOEFF /2.359,1.800,1.416,1.256,0.886 /
28.          DATA MAXWAVNO /50,100,500,1000,10000,100000 /
29.          DATA WVMAXCOF / 2.12,2.28,2.61,2.78,3.13,3.47 /
30.  C        ************************************
31.  C
32.  C               INITIALIZATION
33.  C
34.          NP = 0
35.          NI = 0
36.          ND = 0
37.          CAREAWL = 0.0
38.          VIRTMASS = 0.0
39.          ISEASEL = 0
40.          TIME1 = 0.200
41.          TIME2 = 50.0
42.          TIMEDEL = 0.200
43.          WINDV = 0.0
44.          PI2 = PI*2.0
45.          RHOG = RHO*G
46.  C
47.  C               CHECK FOR ON-LINE
48.          IONFLAG = 0
49.          IF (IAMTERM (IDUM) )   IONFLAG = 1
50.  C
51.  C               INPUT DATA
52.  C
53.     100 CONTINUE
54.          WRITE (NLP,9400)
55.  C
56.  C               INPUT NUMBER OF PRESSURE SURFACES
57.          IF (IONFLAG .EQ. 1)   WRITE (NLP,9410)
58.          INPUT NTEST
59.          IF (NTEST .GT. NMAX)   WRITE (NLP,9700) NTEST,NMAX ,   STOP 100
```

```
60.          IF (NTEST .LT. 0)  GO TO 175
61.          IF (NTEST .GE. 0)  NP = NTEST
62.          IF (NP .EQ. 0)  GO TO 200
63.    C               INPUT PRESSURE TERMS
64.          IF (IONFLAG .EQ. 1)  WRITE (NLP,9420)
65.          DO 150  I = 1,NP
66.            INPUT DEPTHP(I),AREA(I)
67.    150 CONTINUE
68.    C               OUTPUT TERMS
69.    175 CONTINUE
70.          IF (IONFLAG .EQ. 1)  GO TO 200
71.          WRITE (NLP,9430) NP
72.          IF (NP .GT. 0)  WRITE (NLP,9440)  (DEPTHP(I),AREA(I), I=1,NP)
73.    C
74.    C               INPUT NUMBER OF INERTIAL COMPONENTS
75.    200 CONTINUE
76.          IF (IONFLAG .EQ. 1)  WRITE (NLP,9450)
77.          INPUT NTEST
78.          IF (NTEST .GT. NMAX)  WRITE (NLP,9700) NTEST,NMAX ;  STOP 200
79.          IF (NTEST .LT. 0)  GO TO 275
80.          IF (NTEST .GE. 0)  NI = NTEST
81.          IF (NI .EQ. 0)  GO TO 300
82.    C               INPUT INERTIAL TERMS
83.          IF (IONFLAG .EQ. 1)  WRITE (NLP,9460)
84.          DO 250  I = 1,NI
85.            INPUT DEPTHI(I),ADDMSC(I),VOLUME(I)
86.    250 CONTINUE
87.    C               OUTPUT TERMS
88.    275 CONTINUE
89.          IF (IONFLAG .EQ. 1)  GO TO 300
90.          WRITE (NLP,9470)  NI
91.          IF (NI .GT. 0)  WRITE (NLP,9480)  (DEPTHI(I),ADDMSC(I),VOLUME(I),
92.        +                                        I=1,NI)
93.    C
94.    C               INPUT NUMBER OF DRAG SURFACES
95.    300 CONTINUE
96.          IF (IONFLAG .EQ. 1)  WRITE (NLP,9490)
97.          INPUT NTEST
98.          IF (NTEST .GT. NMAX)  WRITE (NLP,9700)  NTEST,NMAX ;  STOP 300
99.          IF (NTEST .LT. 0)  GO TO 375
100.         IF (NTEST .GE. 0)  ND = NTEST
101.         IF (ND .EQ. 0)  GO TO 400
102.   C               INPUT DRAG COMPONENTS
103.         IF (IONFLAG .EQ. 1)  WRITE (NLP,9500)
104.   C               SUM DRAG SURFACE COEFF F(DEPTH=0)
105.         SUMDCO = 0.0
106.         DO 350  I = 1,ND
107.           INPUT DEPTHD(I),DAMPC(I),WDRAGC(I)
108.           SUMDCO = SUMDCO + DAMPC(I)
109.   350 CONTINUE
110.   C               OUTPUT TERMS
111.   375 CONTINUE
112.         IF (IONFLAG .EQ. 1)  GO TO 400
113.         WRITE (NLP,9510)  ND
114.         IF (ND .GT. 0)  WRITE (NLP,9520)  (DEPTHD(I),DAMPC(I),WDRAGC(I),
115.       +                                        I = 1,ND)
116.   C
117.   C               INPUT WATER LEVEL CROSS SECTION AREA
118.   C                   (TO SIMPLIFY THE COMPUTATION THIS IS ASSUMED
119.   C                   CONSTANT OVER THE RANGE OF VERTICAL MOTION AT
```

```
120.    C                        THE WATER LINE)
121.        400 CONTINUE
122.            IF (IONFLAG .EQ. 1)  WRITE (NLP,9530)
123.            INPUT CSATEST
124.            IF (CSATEST .LT. 0.0)  GO TO 450
125.            CAREAWL = CSATEST
126.        450 CONTINUE
127.            IF (IONFLAG .NE. 1)  WRITE (NLP,9540)  CAREAWL
128.            HF = CAREAWL*RHOG
129.    C
130.    C                    INPUT VIRTUAL MASS
131.            IF (IONFLAG .EQ. 1)  WRITE (NLP,9550)
132.            INPUT VMTEST
133.            IF (VMTEST .LT. 0.0)  GO TO 550
134.            VIRTMASS = VMTEST
135.        550 CONTINUE
136.            IF (IONFLAG .NE. 1)  WRITE (NLP,9560)  VIRTMASS
137.    C
138.    C                    INPUT TIME RANGE
139.            IF (IONFLAG .EQ. 1)  WRITE (NLP,9570)
140.            READ (NCR,9025)  T1,T2,T3
141.            IF (T1 .LT. 0.0)  GO TO 675
142.            TIME1 = T1
143.            TIME2 = T2
144.            IF (TIME2 .LT. TIME1)  TIME2 = TIME1
145.            IF (T3 .LE. 0.0)  T3 = TIMEDEL
146.            TIMEDEL = T3
147.        675 CONTINUE
148.            IF (IONFLAG .NE. 1)  WRITE (NLP,9580)  TIME1,TIME2,TIMEDEL
149.    C
150.    C                    INPUT WIND VELOCITY FOR SEA STATE
151.        700 CONTINUE
152.            IF (IONFLAG .EQ. 1)  WRITE (NLP,9590)
153.            READ (NCR,9020)  ISTEST,WAVEHT,WAVEPER
154.            IF (ISTEST .GT. 3)  ISTEST = -1
155.            IF (ISTEST .LT. 0)  GO TO 775
156.            ISEASEL = ISTEST
157.            IF (ISEASEL .EQ. 1)  WINDV = WAVEHT
158.            WINDVP4 = WINDV**4
159.            WAVEHTP2 = WAVEHT*WAVEHT
160.            WAVPERP4 = WAVEPER**4
161.        775 CONTINUE
162.            IF (IONFLAG .EQ. 1)  GO TO 800
163.            IF (ISEASEL .EQ. 0)  WRITE (NLP,9600)
164.            IF (ISEASEL .EQ. 1)  WRITE (NLP,9601)  WINDV
165.            IF (ISEASEL .EQ. 2)  WRITE (NLP,9602)  WAVEHT,WAVEPER
166.            IF (ISEASEL .EQ. 3)  WRITE (NLP,9603)  WAVEHT,WAVEPER
167.    C
168.    C                    CHECK OUTPUT OPTIONS
169.        800 CONTINUE
170.            IF (IONFLAG .EQ. 1)  WRITE (NLP,9605)
171.            READ (NCR,9000)  IL,IP,RMIN,RMAX
172.            ILIST = 0
173.            IF (IL .EQ. 1HY)  ILIST = 1
174.            IPLOT = 0
175.            IF (IP .EQ. 1HY)  IPLOT =1
176.            IF (IPLOT .EQ. 1)  CALL PLOTINIT
177.    C
178.    C                COMPUTE RESPONSE AND PHASE COMPONENT ON TIME ITERATION
179.    C
```

```
180.        900 CONTINUE
181.            IF (ILIST .EQ. 1)  WRITE (NLP,9610)
182.            RRSINTG = 0.0
183.            SINTG = 0.
184.            DO 2000  TIME = TIME1,TIME2,TIMFDEL
185.              FREQ = PI2*10000000000.0
186.              IF (TIME .NE. 0.0)  FREQ = PI2/TIME
187.              FREQP4 = FREQ**4
188.              FREQP5 = FREQ*FREQP4
189.              EXPTERM = -FREQ*FREQ/G
190.    C
191.    C                      SUM PRESSURE COMPONENTS
192.            SUMP = 0.0
193.            REPEAT 1100, FOR I = (1,NP)
194.              SUMP = SUMP + RHOG*AREA(I)*EXP(EXPTERM*DEPTHP(I))
195.       1100  CONTINUE
196.    C                      SUM INERTIAL COMPONENTS
197.            SUMI = 0.0
198.            REPEAT 1200, FOR I = (1,NI)
199.              SUMI = SUMI + ADDMSC(I)*VOLUME(I)*EXP(EXPTERM*DEPTHI(I))
200.       1200  CONTINUE
201.            SUMI = RHO*FREQ*FREQ*SUMI
202.    C
203.    C                      SUM DRAG COMPONENTS
204.            SUMD = 0.0
205.            REPEAT 1300, FOR I = (1,ND)
206.              SUMD = SUMD + WDRAGC(I)*EXP(EXPTERM*DEPTHD(I))
207.       1300  CONTINUE
208.            SUMD = FREQ*FREQ*SUMD
209.            SUMDC = FREQ*FREQ*SUMDCO
210.    C
211.    C                      COMPUTE RESPONSE AMPLITUDE OPERATOR
212.            RNUM = (SUMP - SUMI)**2 + SUMD**2
213.            RDENOM = (RF - VIRTMASS*FREQ*FREQ)**2 + SUMDC**2
214.            RAO = SQRT(RNUM/RDENOM)
215.    C
216.    C                      COMPUTE PHASE BETWEEN FORCE AND HEAVE
217.            PHI = RTOD*ATAN2(-SUMDC,(RF - VIRTMASS*FREQ*FREQ) )
218.    C
219.    C                      COMPUTE PHASE BETWEEN FORCE AND WAVE
220.            SIGMA = RTOD*ATAN2(SUMD,(SUMP - SUMI) )
221.    C
222.    C                      COMPUTE PHASE BETWEEN WAVE AND HEAVE
223.            THETA = SIGMA + PHI
224.    C
225.    C                      GET SEA SPECTRA
226.            CALL SEASPEC
227.    C
228.    C                      COMPUTE RESPONSE AND INTEGRATE
229.            RRS = RAO*RAO*S
230.            IF (TIME .LE. TIME1)  GO TO 1400
231.            DELF = FREQLAST - FREQ
232.            RRSINTG = (RRS + RRSLAST)*0.50*DELF + RRSINTG
233.            SINTG = (S + SLAST)*0.50*DELF + SINTG
234.       1400  CONTINUE
235.            RRSLAST = RRS
236.            SLAST = S
237.            FREQLAST = FREQ
238.    C
239.    C                      OUTPUT LIST IF IT WAS SELECTED
```

```
240.              IF (ILIST .LE. 0)  GO TO 1500
241.              WRITE (NLP,9615)  TIME,FREQ,RAO,SIGMA,PHI,THETA,S
242.      C
243.      C                    CHECK FOR PLOT
244.     1500   CONTINUE
245.              IF (IPLOT .LE. 0)  GO TO 2000
246.              CALL PLOT3 (' ',RAO,TIME,1)
247.      2000 CONTINUE
248.      C
249.      C              GET STATISTICS
250.      C
251.      C                    COMPUTE ROOT MEAN SQUARE OF WAVE
252.            RMS = SQRT(SINTG)
253.            PROBAMP = 0.707*RMS
254.      C                    COMPUTE AVERAGE WAVE HEIGHT
255.            DO 2300 I = 1,5
256.              AVRESPNS(I) = AVRCOEFF(I)*RMS
257.     2300 CONTINUE
258.      C                    COMPUTE MAXIMUM WAVE AMPLITUDES
259.            DO 2400 I = 1,6
260.              HEVMAXHT(I) = WVMAXCOF(I)*RMS
261.     2400 CONTINUE
262.            WRITE (NLP,9635)  RMS
263.            WRITE (NLP,9640)  PROBAMP
264.            WRITE (NLP,9645)  (FRACAMPS(I),AVRESPNS(I),
265.          +                   MAXWAVNO(I),HEVMAXHT(I), I=1,5),
266.          +                   MAXWAVNO(6),HEVMAXHT(6)
267.      C
268.      C                    COMPUTE ROOT MEAN SQUARE OF RESPONSE
269.            RMS = SQRT(HRSINTG)
270.            PROBAMP = 0.707*RMS
271.      C                    COMPUTE AVERAGE RESPONSE OF HEAVE
272.            DO 2500 I = 1,5
273.              AVRESPNS(I) = AVRCOEFF(I)*RMS
274.     2500 CONTINUE
275.      C                    COMPUTE MAXIMUM AMPLITUDES OF HEAVE
276.            DO 2600 I = 1,6
277.              HEVMAXHT(I) = WVMAXCOF(I)*RMS
278.     2600 CONTINUE
279.            WRITE (NLP,9620)  RMS
280.            WRITE (NLP,9625)  PROBAMP
281.            WRITE (NLP,9630)  (FRACAMPS(I),AVRESPNS(I),
282.          +                   MAXWAVNO(I),HEVMAXHT(I), I=1,5),
283.          +                   MAXWAVNO(6),HEVMAXHT(6)
284.      C
285.      C              CHECK FOR PLOT
286.      C
287.            IF (IPLOT .LE. 0)  GO TO 3000
288.            CALL PLOTHEAV
289.     3000 CONTINUE
290.            IF (IONFLAG .EQ. 1)  WRITE (NLP,9655)
291.            READ (NCR,9015,END=8000)  IEND
292.            IF (IEND .EQ. 1HY)  GO TO 100
293.      C
294.      C
295.      C
296.     8000 CONTINUE
297.            STOP
298.      C
299.     9000 FORMAT (A1,1X,A1,1X,2E.0)
```

```
300.      9015 FORMAT (A1)
301.      9020 FORMAT (I,F.0,F.0)
302.      9025 FORMAT (3F.0)
303.   C
304.      9400 FORMAT (1H1,'   HEAVE RESPONSE ANALYSIS PROGRAM'/
305.     +              '            ALL DEPTHS ARE POSITIVE'/)
306.      9410 FORMAT (/' INPUT THE NUMBER OF PRESSURE SURFACES 0')
307.      9420 FORMAT (/' ENTER FOR EACH SURFACE'/
308.     +              ' DEPTH (FT), AREA (SQ FT) (= AREA FOR DOWNWARD FORCE)')
309.      9430 FORMAT (/' NUMBER OF PRESSURE SURFACES =',I5)
310.      9440 FORMAT (/' DEPTH (FT), AREA (SQ FT) (= AREA FOR DOWNWARD FORCE)'/
311.     +              (F10.3,2X,F8.2) )
312.      9450 FORMAT (/' INPUT THE NUMBER OF INERTIAL COMPONENTS 0')
313.      9460 FORMAT (/' ENTER FOR EACH COMPONENT'/
314.     +              ' DEPTH (FT), ADDED MASS COEFF, VOLUME (CU FT)' )
315.      9470 FORMAT (/' NUMBER OF INERTIAL COMPONENTS =',I5)
316.      9480 FORMAT (/'  DEPTH (FT)  ADDED MASS COEFF   VOLUME (CU FT)'/
317.     +              (F10.3,6X,F10.3,7X,F12.3) )
318.      9490 FORMAT (/' INPUT THE NUMBER OF DRAG SURFACES 0')
319.      9500 FORMAT (/' ENTER FOR EACH SURFACE'/
320.     +              ' DEPTH (FT), DAMPING COEFF, WAVE DRAG COEFF'
321.     +              ' (LBF/(FT/SEC)/(RAD/SEC))' )
322.      9510 FORMAT (/' NUMBER OF DRAG SURFACES =',I5)
323.      9520 FORMAT (/'  DEPTH (FT)  DAMPING COEFF  WAVE DRAG COEFF'
324.     +              ' (LBF/(FT/SEC)/(RAD/SEC))' /
325.     +              (F10.3,3X,F11.3, 5X,F11.3) )
326.      9530 FORMAT (/'  ENTER CROSS SECTION AREA AT SURFACE (SQ FT) 0')
327.      9540 FORMAT (/'  CROSS SECTIONAL AREA AT SURFACE =',F10.4,' SQ FEET')
328.      9550 FORMAT (/'  ENTER VIRTUAL MASS (SLUGS) 0')
329.      9560 FORMAT (/'  VIRTUAL MASS =', F11.2, ' SLUGS')
330.      9570 FORMAT (/'  ENTER START, END, INCREMENT OF PERIOD RANGE (SEC) 0')
331.      9580 FORMAT (/'  PERIOD RANGE, IN SECONDS    START    END    DELTA'/
332.     +              26X,F8.3,F8.3,2X,F8.3)
333.      9590 FORMAT (/'  ENTER SEA SPECTRUM TYPE AND PARAMETERS'
334.     +        /'        = 1.0          0 '
335.     +        /'      PIERSON-MOSKOWITZ 1, WIND SPEED (KNOTS)'
336.     +        /'      BRETSCHNEIDER    2, SIGNIF WAVE HT (FT), SIGNIF'
337.     +        ' WAVE PERIOD (SEC)'
338.     +        /'      I.S.S.C.         3, SIGNIF WAVE HT (FT), SIGNIF'
339.     +        ' WAVE PERIOD (SEC)'          )
340.      9600 FORMAT (/'  SEA SPECTRUM = 1.0')
341.      9601 FORMAT (/'  PIERSON-MOSKOWITZ SEA SPECTRUM '
342.     +        /'      WIND SPEED = ',F10.3,' KNOTS')
343.      9602 FORMAT (/'  BRETSCHNEIDER SEA SPECTRUM'
344.     +        /'      SIGNIFICANT WAVE HT = ',F10.3,' FEET'
345.     +        /'      SIGNIFICANT WAVE PERIOD = ',F10.3,' SEC')
346.      9603 FORMAT (/'  I.S.S.C. SEA SPECTRUM'
347.     +        /'      SIGNIFICANT WAVE HT = ',F10.3,' FEET'
348.     +        /'      SIGNIFICANT WAVE PERIOD = ',F10.3,' SEC')
349.      9605 FORMAT (/' ENTER Y OR N FOR A LISTING, PLOT OF RESPONSE  '/
350.     +        '       FOR PLOT, YOU MAY ALSO ENTER RAD MIN,MAX 0')
351.      9610 FORMAT (1H1/'  PERIOD   ANG FREQ    RAO   W-F PHASE F-H PHASE '
352.     +        'W-H PHASE   AMP SPEC ')
353.      9615 FORMAT (F10.3,E10.3,F10.3,F10.3,F10.3,F10.3,F10.3)
354.      9620 FORMAT (////////'  RMS OF RESPONSE SPECTRUM =',F10.3,' FEET')
355.      9625 FORMAT (/'  PROBABLE AMPLITUDE '/
356.     +        '             OF HEAVE RESPONSE =',F10.3,' FEET')
357.      9630 FORMAT (//
358.     +        /' FRACTION OF      AVERAGE   !          EXPECTED  '
359.     +        /'  LARGEST        HEAVE     !           HEAVE    '
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360.          +          /' AMPLITUDES        AMPLITUDE I    NUMBER      MAXIMUM     '
361.          +          /' CONSIDERED        RESPONSE  I    OF WAVES    AMPLITUDE   '
362.          +          /' -----------       --------- I    --------    ---------   '
363.          +          5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
364.          +          /                     T28,'I', T33,I6, T43,F9.3 //)
365.     9635 FORMAT (1H1////' RMS OF WAVE SPECTRUM =',F10.3,' FEET')
366.     9640 FORMAT (/' PROBABLE AMPLITUDE '/
367.          +          '       OF WAVE              =',F10.3,' FEET')
368.     9645 FORMAT (//
369.          +          /' FRACTION OF                I                   EXPECTED  '
370.          +          /'   LARGEST        AVERAGE   I                     WAVE    '
371.          +          /' AMPLITUDES        WAVE     I    NUMBER      MAXIMUM     '
372.          +          /' CONSIDERED        AMPLITUDE I    OF WAVES    AMPLITUDE   '
373.          +          /' -----------       --------- I    --------    ---------   '
374.          +          5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
375.          +          /                     T28,'I', T33,I6, T43,F9.3     )
376.     9655 FORMAT (/' DO YOU WANT ANOTHER CASE 0')
377.    C
378.     9700 FORMAT (' *** NUMBER OF ENTRIES IS GREATER THAN ARRAY SIZE ALLOWS'
379.          +          /14X,I8,20X,I7,
380.          +          /'       THE PROGRAM TERMINATES' )
381.    C
382.    C          **************************************************************
383.    C
384.          SUBROUTINE SEASPEC
385.          IF (ISEASEL .EQ. 0)  S = 1.0 ;  RETURN
386.          GO TO  (4100,4200,4300), ISEASEL
387.    C               PIERSON - MOSKOWITZ
388.     4100 CONTINUE
389.          S = 135.0/FREQP5*EXP(-97000.0/(FREQP4*WINDVP4))
390.    C                    CORRECT FOR DOUBLE HEIGHT SPECTRUM
391.          S = S/8.0
392.          RETURN
393.    C               BRETSCHNEIDER
394.     4200 CONTINUE
395.          S = 4200.0*WAVEHTP2/(WAVPERP4*FREQP5)*
396.          +      EXP(-1050.0/(WAVPERP4*FREQP4))
397.    C                    CORRECT FOR DOUBLE HEIGHT SPECTRUM
398.          S = S/8.0
399.          RETURN
400.    C               I.S.S.C.
401.     4300 CONTINUE
402.          S = 2760.0*WAVEHTP2/(WAVPERP4*FREQP5)*
403.          +      EXP(-630.0/(WAVPERP4*FREQP4))
404.    C                    CORRECT FOR DOUBLE HEIGHT SPECTRUM
405.          S = S/8.0
406.          RETURN
407.    C
408.    C
409.    C          **************************************************************
410.    C
411.          SUBROUTINE PLOTINIT
412.    C               THIS SUBROUTINE IS USED TO INITIALIZE A LINE PRINTER
413.    C               PLOT OF THE HEAVE RESPONSE.
414.    C
415.          DIMENSION IPLOTBUF(3300)
416.    C
417.          DATA IBUFSIZE /3300/
418.    C
419.    C
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420.        IF (RMAX .GT. RMIN)  GO TO 1490
421.        RMAX = 100*RMIN
422.        IF (RMAX .EQ. 0.0)  RMAX = 5.0
423.   1490 CONTINUE
424.        RDEL = RMAX - RMIN
425.        RMAX = IFIX(RDEL/5.0 + 0.999)*5.0 + RMIN
426.  C
427.        NLINES = (TIME2 - TIME1)/TIMEDEL
428.        NBARS = (NLINES + 9.1)/10.0
429.        TMAX = TIME1 + NBARS*10.0*TIMEDEL
430.        NLINES = (TMAX - TIME1)/TIMEDEL + 1.0
431.        IF (NLINES .LT. IBUFSIZE/13)  GO TO 5000
432.        IPLOT = 0
433.        WRITE (NLP,9710)
434.        RETURN
435.  C
436.   5000 CONTINUE
437.        CALL PLOT1 (NBARS,10,5,10)
438.        CALL PLOT2 (IPLOTBUF,RMIN,RMAX,TMAX,TIME1)
439.        RETURN
440.  C
441.   9710 FORMAT (/' ***** THE PLOT BUFFER IS NOT LARGE ENOUGH '
442.       +        /'        FOR THE PERIOD RANGE SPECIFIED '
443.       +        /'        THE PLOT IS SUPPRESSED')
444.  C
445.  C
446.  C        ******************************************************
447.  C
448.        SUBROUTINE PLOTHEAV
449.  C
450.  C           THIS SUBROUTINE IS USED TO OUTPUT THE LINE PRINTER
451.  C            PLOT OF THE HEAVE RESPONSE.
452.  C
453.        CALL PLOT5 (3,33,'HEAVE RESPONSE AMPLITUDE OPERATOR')
454.        CALL PLOT4 (14,' PERIOD (SEC) ')
455.        CALL PLOT7 (10)
456.        RETURN
457.        END
```

APPENDIX VIII

Roll Program Listing

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1.    C                          PROGRAM ROLLRAO
2.    C
3.    C         VERSION 1.0          SEP, 1976          R. GOLDSMITH
4.    C
5.    C         THIS PROGRAM IS USED TO COMPUTE THE ROLL RESPONSE
6.    C         AMPLITUDE OPERATOR, AND ASSOCIATED PHASE ANGLES, FOR
7.    C         SPAR TYPE BUOY SYSTEMS.
8.    C
9.    C         CURRENT VERSION RESTRICTS DESIGN TO CYLINDRICAL AND
10.   C         TRIANGULAR BODIES ON END, AND RECTANGULAR PLATES.
11.   C
12.   C
13.           LOGICAL IAMTERM
14.   C
15.           DIMENSION FRACAMPS (5), AVRCOEFF(5), AVRESPNS(5)
16.           DIMENSION MAXWAVNO(6), WVMAXCOF(6), ROLLMAX(6)
17.   C
18.           COMMON / IODEV / NCR,NLP
19.           COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
20.           COMMON / SEASTATE / ISEASEL,WINDV,WAVEHT,WAVEPER,
21.          +                    WINDVP4,WAVEHTP2,WAVPERP4
22.           COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
23.          +                DENSITY(50),DISTCGK(50),FRACNORM(50)
24.           COMMON / BOUTS / VOLUME(50),WEIGHT(50)
25.           COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
26.          +                    DEPTHB(50),DEPTHT(50)
27.           COMMON / CONSTANT / PI,RTOD,RHO,G
28.           COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
29.          +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
30.           COMMON / COEFS / DRAG(5),COEFM(5)
31.           COMMON / MOMENTS / BUOYMI,ADDMI,VIRTINRT,WATERIM,BUOYMR,
32.          +                   BUOYMDT,BUOYMD,WATERMD,DAMPM
33.           COMMON / OUTPUTS / ILIST,IPLOT,RMIN,RMAX
34.   C
35.   C
36.           DATA NCR,NLP /105,108/
37.           DATA PI,RTOD /3.141592,57.2958/
38.           DATA RHO,G / 1.99035, 32.174/
39.   C              MAXIMUM ARRAY SIZES
40.           DATA NPMAX /50/
41.   C              DRAG COEFFICIENTS FOR CYLINDER AND PLATE
42.           DATA DRAG / 1.2, 1.2, 1.5, 0.0, 0.0 /
43.   C              ADDED MASS COEFFICIENTS FOR CYLINDER AND PLATE
44.           DATA COEFM / 1.0, 1.0, 1.0, 0.0, 0.0 /
45.   C              STATISTICAL COEFFICIENTS
46.           DATA FRACAMPS /0.01,0.10,0.3333,0.50,1.0/
47.           DATA AVRCOEFF /2.359,1.800,1.416,1.256,0.886/
48.           DATA MAXWAVNO /50,100,500,1000,10000,100000/
49.           DATA WVMAXCOF /2.12,2.28,2.61,2.78,3.13,3.47/
50.   C
51.   C         *********************** ****************
52.   C
53.   C              INITIALIZATION
54.   C
55.           PI2 = PI*2.0
56.           RHOG = RHO*G
57.           TIME1 = 0.2
58.           TIME2 = 50.0
59.           TIMEDEL = 0.200
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60.          ISEASEL = 0
61.          WINDV = 0.0
62.          WAVEHT = 0.0
63.          WAVEPER = 0.0
64.          HWL = 0.0
65.          AVERGAMP = 3.0
66.          THETABAR = 11.5/RTBD
67.          NPARTS = 0
68.   C
69.      100 CONTINUE
70.   C
71.   C                CHECK FOR ON LINE AND INPUT MODE
72.          IONFLAG = 0
73.      150 CONTINUE
74.          WRITE (NLP,9400)
75.          IF (IAMTERM (IDUM) ) IONFLAG = 1 ;  CALL TINPUT
76.          IF (IONFLAG .EQ. 0)  CALL BINPUT
77.          IF (IPLOT .EQ. 1)  CALL PLOTINIT
78.   C
79.   C                COMPUTE TOTAL BUOY WEIGHT,    DISTANCE FROM KEEL TO
80.   C                CENTER OF GRAVITY,    DEPTH OF CG
81.   C
82.          BUOYWGT = 0.0
83.          SUMT = 0.0
84.          DO 400  I = 1,NPARTS
85.            BUOYWGT = BUOYWGT + WEIGHT(I)
86.            SUMT = SUMT + WEIGHT(I)*DISTCGK(I)
87.      400 CONTINUE
88.          BUOYCGK = SUMT/BUOYWGT
89.          DEPTHCG = DEPTHK - BUOYCGK
90.   C
91.   C                COMPUTE THE PART BASIC MOMENT OF INERTIA CONTRIBUTION
92.   C
93.          SUMT = 0.0
94.          DO 500  I = 1,NPARTS
95.            CALL BODYMI (ISHAPE(I),HEIGHT(I),WIDTH(I),THICK(I),PINERT)
96.   C                FOR THE BODY ABOUT ITS OWN AXIS
97.            PMI = WEIGHT(I)*PINERT/G
98.   C                ABOUT THE CG
99.            BMICOMP = (WEIGHT(I)/G)*(DISTCGK(I) - BUOYCGK)**2
100.           SUMT = SUMT + PMI + BMICOMP
101.     500 CONTINUE
102.         BUOYMI = SUMT
103.  C
104.  C                GET DISPLACEMENT CONTRIBUTIONS
105.  C
106.         CALL DISPLACE
107.  C
108.         BUOYCGCB = BUOYCBK - BUOYCGK
109.  C
110.         WRITE (NLP,9405)  BUOYWGT,WDISPLAC
111.  C                COMPUTE DISTANCE TO METACENTER FROM CB
112.  C
113.         SURFINRT = PI*RWL**4/4.0
114.         BUOYCBM = SURFINRT*RHOG/WDISPLAC
115.  C
116.  C                COMPUTE RIGHTING ARM, GM
117.  C
118.         BUOYCGM = BUOYCGCB + BUOYCBM
119.  C
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120.    C                      CHECK FOR STABILITY
121.               IF (BUOYCGM .LT. 0.0)   WRITE (NLP,9700)   BUOYCBK+BUOYCBM,
122.            +                                            BUOYCGK )
123.            +                      STOP 550
124.    C
125.    C             ********************************************
126.    C
127.    C             COMPUTE RIGHTING MOMENT TERM
128.    C
129.            BUOYMR = BUOYWGT*BUOYCGM
130.            BUOYMRG = -BUOYMR/G
131.    C
132.    C             COMPUTE NATURAL PERIOD OF ROLL
133.    C                  FIRST GET VIRTUAL MOMENT
134.            VIRTINRT = BUOYMI + ADDMI
135.            PERIODO = 2.0*PI*SQRT (VIRTINRT/BUOYMR)
136.            WRITE (NLP,9410)  PERIODO
137.    C
138.    C             MAKE ASSUMPTION OF UNIT AMPLITUDE
139.    C             AND START FREQUENCY ANALYSIS
140.    C
141.            IF (ILIST .EQ. 1)  WRITE (NLP,9450)
142.            RRSINTG = 0.0
143.            SINTG = 0.0
144.    X       OUTPUT BUOYMI,BUOYMR,BUOYMRG
145.    C             **************************************************
146.    C             NOTE: THE CALL TO BUOYDAMP IS PLACED HERE TO
147.    C                   SIMPLIFY THE COMPUTATION. THE CONSTANT TERM
148.    C                   IS COMPUTED HERE AND THE FREQ IS MULTIPLIED
149.    C                   IN AT THE BEGINNING OF THE FREQUENCY ITERATION.
150.    C             **************************************************
151.            CALL BUOYDAMP
152.            DO 2000  TIME = TIME1,TIME2,TIMEDEL
153.              FREQ = PI2*1.0E+10
154.              IF (TIME .NE. 0.0)  FREQ = PI2/TIME
155.              WAVEN = FREQ*FREQ/G
156.    C
157.    C             *** COMPUTE DAMPING MOMENTS
158.    C
159.    C                   SEE NOTE ABOVE
160.            BUOYMD = BUOYMDT*FREQ
161.            CALL WATERDAMP
162.    C
163.    C             *** COMPUTE WAVE INERTIA MOMENTS
164.    C
165.            CALL WATRINRT
166.    X       OUTPUT BUOYMD,WATERMD,WATERIM
167.    C
168.    C
169.            ETEM = BUOYMRG + WATERIM
170.            FTEM = WATERMD
171.    C
172.    C                   SET MAIN COMPONENT
173.            E = ETEM*FREQ*FREQ
174.            F = FTEM*FREQ
175.    C                   PHASE BETWEEN WAVE AND TORQUE
176.            SIGMA = RTOD*ATAN2 (-E,F)
177.    C
178.    C                   EXCITING TORQUE
179.            T = SQRT (E*E + F*F)
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180.    C
181.    C                      PHASE ANGLE BETWEEN TORQUE AND ROLL
182.          ATEM = BUBYMO*FREQ
153.          BTEM = BUBYMR - VIRTINRT*FREQ*FREQ
184.          PHI = RTOD*ATAN2 (-ATEM,BTEM)
185.    C
186.    C                      ROLL RAO
187.    C
188.          ROLLRAO = T/SQRT (ATEM*ATEM + BTEM*BTEM)
189.    C
190.    C                      PHASE ANGLE BETWEEN WAVE AND ROLL
191.    C
192.          THETA = SIGMA + PHI
193.    C
194.    C                      GET SEA SPECTRA
195.          CALL SEASPEC
196.    C
197.    C                      COMPUTE RESPONSE AND INTEGRATE
198.          RRS = ROLLRAO*ROLLRAO*S
199.          IF (TIME .LE. TIME1)  GO TO 1400
200.          DELF = FREQLAST - FREQ
201.          RRSINTG = (RRS + RRSLAST)*0.50*DELF + RRSINTG
202.          SINTG = (S + SLAST)*0.50*DELF + SINTG
203.    1400  CONTINUE
204.          RRSLAST = RRS
205.          SLAST = S
206.          FREQLAST = FREQ
207.    C
208.    C                      OUTPUT LIST IF IT WAS SELECTED
209.          IF (ILIST .LE. 0)  GO TO 1500
210.          WRITE (NLP,9455)  TIME,FREQ,ROLLRAO*RTOD,SIGMA,PHI,THETA,S
211.    C
212.    C                      CHECK FOR PLOT
213.    1500  CONTINUE
214.          IF (IPLOT .LE. 0)  GO TO 2000
215.          CALL PLOT3 ('*',ROLLRAO*RTOD,TIME,1)
216.    C
217.    2000  CONTINUE
218.    C
219.    C              GET STATISTICS
220.    C
221.    C                      COMPUTE ROOT MEAN SQUARE OF WAVE
222.          RMS = SQRT(SINTG)
223.          PROBAMP = 0.707*RMS
224.    C                      COMPUTE AVERAGE WAVE HEIGHT
225.          DO 2300 I = 1,5
226.             AVRESPNS(I) = AVRCOEFF(I)*RMS
227.    2300  CONTINUE
228.    C                      COMPUTE MAXIMUM WAVE AMPLITUDES
229.          DO 2400 I = 1,6
230.             ROLLMAX(I) = WVMAXCOF(I)*RMS
231.    2400  CONTINUE
232.          WRITE (NLP,9500)  RMS
233.          WRITE (NLP,9505)  PROBAMP
234.          WRITE (NLP,9510)  (FRACAMPS(I),AVRESPNS(I),
235.         +                   MAXWAVNO(I),ROLLMAX(I), I=1,5),
236.         +                   MAXWAVNO(6),ROLLMAX(6)
237.    C
238.    C                      COMPUTE ROOT MEAN SQUARE OF RESPONSE
239.          RMS = SQRT(RRSINTG)
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240.          PROBAMP = 0.707*RMS
241.    C                   COMPUTE AVERAGE RESPONSE OF ROLL
242.          DO 2500 I = 1,5
243.             AVRESPNS(I) = AVRCOEFF(I)*RMS*RTOD
244.     2500 CONTINUE
245.    C                   COMPUTE MAXIMUM AMPLITUDES OF ROLL
246.          DO 2600 I = 1,6
247.             ROLLMAX(I) = WVMAXCOF(I)*RMS*RTOD
248.     2600 CONTINUE
249.          WRITE (NLP,9515)   RMS*RTOD
250.          WRITE (NLP,9520)   PROBAMP*RTOD
251.          WRITE (NLP,9525)  (FRACAMPS(I),AVRESPNS(I),
252.         +                   MAXWAVNO(I),ROLLMAX(I), I=1,5),
253.         +                   MAXWAVNO(6),ROLLMAX(6)
254.    C
255.    C           CHECK FOR PLOT
256.    C
257.          IF (IPLOT .LE. 0)  GO TO 3000
258.          CALL PLOTROLL
259.     3000 CONTINUE
260.    C
261.          IF (IONFLAG .EQ. 1)  WRITE (NLP,9485)
262.          READ (NCR,9085,END=8000)  IEND
263.          IF (IEND .EQ. 1HY)  GO TO 150
264.    C
265.     8000 CONTINUE
266.          STOP
267.    C
268.     9085 FORMAT (A1)
269.    C
270.     9400 FORMAT (1H1,'          ROLL RESPONSE ANNALYSIS PROGRAM'/
271.         +           '               ALL DEPTHS ARE POSITIVE '/)
272.     9405 FORMAT (/'  CHECK FOR BUOYANCY      BUOY WEIGHT = ',F10.1,' LBS'/
273.         +           '                     WATER DISPLACED = ',F10.1,' LBS')
274.     9410 FORMAT (/' NATURAL PERIOD =',F8.3,' SECONDS')
275.     9450 FORMAT (1H1/'     RAO IS IN DEGREES/FOOT OF WAVE AMPLITUDE'/
276.         +          '  PERIOD   ANG FREQ     RAO    W-T PHASE T-R PHASE '
277.         +          'W-R PHASE  AMP SPEC ')
278.     9455 FORMAT (F10.3,E10.3,F10.3,F10.3,F10.3,F10.3,F10.3)
279.     9485 FORMAT (/' DO YOU WANT ANOTHER CASE 0')
280.     9500 FORMAT (1H1/////' RMS OF WAVE SPECTRUM =',F10.3,' FEET')
281.     9505 FORMAT (/' PROBABLE AMPLITUDE '/
282.         +          '     OF WAVE          =',F10.3,' FEET')
283.     9510 FORMAT (//
284.         +     /' FRACTION OF               I              EXPECTED '
285.         +     /'   LARGEST       AVERAGE    I                WAVE   '
286.         +     /' AMPLITUDES       WAVE     I    NUMBER      MAXIMUM '
287.         +     /' CONSIDERED     AMPLITUDE I   OF WAVES    AMPLITUDE '
288.         +     /' -----------    --------- I   --------    --------- '
289.         +     5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
290.         +     /              T24,'I', T33,I6, T43,F9.3   )
291.     9515 FORMAT (////////'  RMS OF RESPONSE SPECTRUM =',F10.3,' DEG    ')
292.     9520 FORMAT (/'  PROBABLE AMPLITUDE '/
293.         +          '        OF ROLL RESPONSE  =',F10.3,' DEG    ')
294.     9525 FORMAT (//
295.         +     /' AMPLITUDES OF ROLL ARE IN DEGREES'/
296.         +     /' FRACTION OF      AVERAGE   I              EXPECTED '
297.         +     /'   LARGEST         ROLL     I                ROLL   '
298.         +     /' AMPLITUDES     AMPLITUDE I    NUMBER      MAXIMUM '
299.         +     /' CONSIDERED     RESPONSE  I   OF WAVES    AMPLITUDE '
```

```
300.          +       /' ----------    ---------- !   --------   ----------  !
301.          +       5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
302.          +       /                        T28,'I', T33,I6, T43,F9.3 //)
303.    C
304.    9700 FORMAT (//'    *** STOP EVERYTHING - THIS BUOY WILL ROLL OVER!/
305.          +          !              THE CENTER OF GRAVITY IS ABOVE THE META!/
306.          +          !          - CENTER.   MC ABOVE KEEL = ',F6.2,' FEET!/
307.          +          !                       CG ABOVE KEEL = ',F6.2,' FEET!)
308.    C
309.    C
310.    C       ***************************************************************
311.    C
312.          SUBROUTINE SEASPEC
313.          IF (ISEASEL .EQ. 0)  S = 1.0 !  RETURN
314.          FREQP4 = FREQ**4
315.          FREQP5 = FREQP4*FREQ
316.          GO TO  (4100,4200,4300), ISEASEL
317.    C            PIERSON - MOSKOWITZ
318.    4100 CONTINUE
319.          S = 135.0/FREQP5*EXP(-97000.0/(FREQP4*WINDVP4))
320.    C                   CORRECT FOR DOUBLE HEIGHT SPECTRUM
321.          S = S/8.0
322.          RETURN
323.    C            BRETSCHNEIDER
324.    4200 CONTINUE
325.          S = 4200.0*WAVEHTP2/(WAVPERP4*FREQP5)*
326.          +        EXP(-1050.0/(WAVPERP4*FREQP4))
327.    C                   CORRECT FOR DOUBLE HEIGHT SPECTRUM
328.          S = S/8.0
329.          RETURN
330.    C            I.S.S.C.
331.    4300 CONTINUE
332.          S = 2760.0*WAVEHTP2/(WAVPERP4*FREQP5)*
333.          +        EXP(-630.0/(WAVPERP4*FREQP4))
334.    C                   CORRECT FOR DOUBLE HEIGHT SPECTRUM
335.          S = S/8.0
336.          RETURN
337.    C
338.    C
339.    C       ***************************************************************
340.    C
341.          SUBROUTINE PLOTINIT
342.    C            THIS SUBROUTINE IS USED TO INITIALIZE A LINE PRINTER
343.    C            PLOT OF THE ROLL RESPONSE.
344.    C                       SIZE LIMITED FOR ON-LINE USE ONLY
345.    C
346.          DIMENSION IPLOTBUF(1300)
347.    C
348.          DATA IBUFSIZE / 1300/
349.    C
350.    C
351.          IF (RMAX .GT. RMIN)  GO TO 1490
352.          RMAX = 100*RMIN
353.          IF (RMAX .EQ. 0.0)  RMAX = 10.0
354.    1490 CONTINUE
355.          RDEL = RMAX - RMIN
356.          RMAX = IFIX(RDEL/5.0 + 0.999)*5.0 + RMIN
357.    C
358.          NLINES = (TIME2 - TIME1)/TIMEDEL
359.          NBARS = (NLINES + 9.1)/10.0
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360.          TMAX = TIME1 + NBARS*10.0*TIMEDEL
361.          NLINES = (TMAX - TIME1)/TIMEDEL + 1.0
362.          IF (NLINES .LT. IBUFSIZE/13)  GO TO 5000
363.          IPLOT = 0
364.          WRITE (NLP,9710)
365.          RETURN
366.      C
367.   5000 CONTINUE
368.          CALL PLOT1 (NBARS*10,5,10)
369.          CALL PLOT2 (IPLOTBUF,RMIN,RMAX,TMAX,TIME1)
370.          RETURN
371.      C
372.   9710 FORMAT (/' ***** THE PLOT BUFFER IS NOT LARGE ENOUGH '
373.          +        /'        FOR THE PERIOD RANGE SPECIFIED '
374.          +        /'          THE PLOT IS SUPPRESSED')
375.      C
376.      C
377.      C       ********************************************************
378.      C
379.          SUBROUTINE PLOTROLL
380.      C
381.      C           THIS SUBROUTINE IS USED TO OUTPUT THE LINE PRINTER
382.      C           PLOT OF THE ROLL RESPONSE.
383.      C
384.          CALL PLOT5 (3,33,'ROLL RESPONSE AMPLITUDE OPERATOR')
385.          CALL PLOT4 (14,' PERIOD (SEC) ')
386.          CALL PLOT5 (2,28,' DEGREES PER FOOT AMPLITUDE ')
387.          CALL PLOT7 (10)
388.          RETURN
389.      C
390.          END
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  1.            SUBROUTINE TINPUT
  2.      C
  3.      C              VERSION 1.0        SEP, 1976        R. GOLDSMITH
  4.      C
  5.      CCCC           THIS ROUTINE INPUTS DATA FOR THE ROLL RAO IN ON-LINE MODE
  6.      C
  7.      C
  8.            COMMON / IODEV / NCR,NLP
  9.            COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
 10.            COMMON / SEASTATE / ISEASEL,WINDV,WAVEHT,WAVEPER,
 11.           +                    WINDVP4,WAVEHTP2,WAVPERP4
 12.            COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
 13.           +                DENSITY(50),DISTCGK(50),FRACNORM(50)
 14.            COMMON / BOUTS / VOLUME(50),HEIGHT(50)
 15.            COMMON / CONSTANT / PI,RTOD,RHO,G
 16.            COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
 17.           +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
 18.            COMMON / OUTPUTS / ILIST,IPLOT,RMIN,RMAX
 19.      C
 20.      C
 21.      C              INPUT TIME AND RANGE
 22.            WRITE (NLP,9400)
 23.            READ (NCR,9000)  T1,T2,T3
 24.            IF (T1 .LT. 0.0) GO TO 200
 25.            TIME1 = T1
 26.            TIME2 = T2
 27.            IF (TIME2 .LT. TIME1)  TIME2 = TIME1
 28.            IF (T3 .LE. 0.0)  T3 = TIMEDEL
 29.            TIMEDEL = T3
 30.      C
 31.      C              SELECT SEA STATE PARAMETERS
 32.        200 CONTINUE
 33.            WRITE (NLP,9405)
 34.            READ (NCR,9005)  ISTEST, WAVEHT, WAVEPER
 35.            IF (ISTEST .GT. 3)  ISTEST = -1
 36.            IF (ISTEST .LT. 0)  GO TO 300
 37.            ISEASEL = ISTEST
 38.            IF (ISEASEL .EQ. 1)  WINDV = WAVEHT
 39.            WINDVP4 = WINDV**4
 40.            WAVEHTP2 = WAVEHT*WAVEHT
 41.            WAVPERP4 = WAVEPER**4
 42.      C
 43.      C              ENTER WATER PLANE RADIUS
 44.        300 CONTINUE
 45.            WRITE (NLP,9410)
 46.            READ (NCR,9010)  RWLTEST
 47.            IF (RWLTEST .LT. 0.0)  GO TO 400
 48.            RWL = RWLTEST
 49.      C
 50.      C              INPUT DEPTH TO KEEL
 51.        400 CONTINUE
 52.            WRITE (NLP,9415)
 53.            READ (NCR,9015)  ZKTEST
 54.            IF (ZKTEST .LT. 0.0)  GO TO 500
 55.            DEPTHK = ZKTEST
 56.      C
 57.      C              ENTER ESTIMATED AVERAGE AMPLITUDE
 58.        500 CONTINUE
 59.            WRITE (NLP,9420)
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60.          READ (NCR,9020)  AMPTEM
61.          IF (AMPTEM .LT. 0.0)  GO TO 600
62.          AVERGAMP = AMPTEM
63.    C
64.    C              ENTER ESTIMATED AVERAGE ROLL
65.     600 CONTINUE
66.          WRITE (NLP,9425)
67.          READ (NCR,9025)  THETATEM
68.          IF (THETATEM .LT. 0.0)  GO TO 700
69.          THETABAR = THETATEM/RTOD
70.    C
71.    C
72.     700 CONTINUE
73.    C
74.    C              DEFINE BUOY
75.    C
76.          N = 0
77.    1000 CONTINUE
78.          WRITE (NLP,9455)
79.          READ (NCR,9055)  NPTEM
80.          IF (NPTEM .LE. 0)  GO TO 2000
81.    C
82.          WRITE (NLP,9460)
83.          N = 1
84.    C
85.    C              LOOP ON ENTRY
86.    1050 CONTINUE
87.          IF (N .GT. NPMAX)  WRITE (NLP,9725) NPMAX ;  GO TO 200
88.          WRITE (NLP,9465)  N
89.          READ (NCR,9065)  IDUM
90.          WRITE (NLP,9470)
91.          READ (NCR,9070)  K,W,H,T,D,X,F
92.    C
93.    C              SET INPUTS FOR CORRECT SHAPE
94.          IF ((K .GT. 5) .OR. (K .LT. 1))  OUTPUT K ;  WRITE (NLP,9700) ;
95.    +                                       GO TO 1050
96.          IF (W .LE. 0.0)  WRITE (NLP,9705) ;  OUTPUT W ;  GO TO 1050
97.          WIDTH(N) = W
98.          IF (H .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT H ;  GO TO 1050
99.          HEIGHT(N) = H
100.         IF (D .LT. 0.0)  WRITE (NLP,9710) ;  OUTPUT D ;  GO TO 1050
101.         DENSITY(N) = D
102.         IF (X .LT. 0.0)  WRITE (NLP,9715) ;  OUTPUT X ;  GO TO 1050
103.         DISTCGK(N) = X
104.         GO TO (1100,1100,1100,1400,1500), K
105.   C
106.   C              CYLINDERS
107.   1100 CONTINUE
108.         IF (T .EQ. -1.0)  T = W*6.0
109.         IF ((K .EQ. 2) .OR. (K .EQ. 3) )  T = W*6.0
110.         IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  GO TO 1050
111.         THICK(N) = T/12.0
112.         FRACNORM(N) = 1.0
113.         ISHAPE(N) = 1
114.         GO TO 1800
115.   C
116.   C              TRIANGLE
117.   1400 CONTINUE
118.         IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  GO TO 1050
119.         THICK(N) = T/12.0
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120.          IF ((F .GT. 1.0) .OR. (F .LT. 0.0))  WRITE (NLP,9720) ,  OUTPUT F
121.        +                                      ,  GO TO 1050
122.          FRACNORM(N) = F
123.          ISHAPE(N) = 2
124.          GO TO 1800
125.    C
126.    C              RECTANGLE
127.     1500 CONTINUE
128.          IF (T .LT. 0.0)  WRITE (NLP,9705) ,  OUTPUT T ,  GO TO 1050
129.          THICK(N) = T/12.0
130.          IF ((F .GT. 1.0) .OR. (F .LT. 0.0))  WRITE (NLP,9720) ,  OUTPUT F
131.        +                                      ,  GO TO 1050
132.          FRACNORM(N) = F
133.          ISHAPE(N) = 3
134.          GO TO 1800
135.    C
136.    C              COMPUTE VOLUME AND WEIGHT
137.     1800 CONTINUE
138.          CALL BODYVOL (ISHAPE(N),H,W,THICK(N),V)
139.          WEIGHT(N) = V*D
140.          VOLUME(N) = V
141.    C
142.    C              CHECK NUMBER OF ENTRIES
143.          IF (N .GE. NPTEM)  GO TO 2000
144.          N = N + 1
145.          GO TO 1050
146.    C
147.    C              PART CHANGE
148.    C
149.     2000 CONTINUE
150.          IF (NPTEM .GT. 0)  NPARTS = NPTEM
151.          NPTEM = 0
152.          NPARTS = MAX (NPARTS,N)
153.          WRITE (NLP,9475)
154.          READ (NCR,9075)  N
155.          IF ((N .GT. 0) .AND. (N .LE. NPARTS + 1))  GO TO 1050
156.    C
157.     3000 CONTINUE
158.          WRITE (NLP,9480)
159.          READ (NCR,9080)  IL,IP,RMIN,RMAX
160.          ILIST = 0
161.          IF (IL .EQ. 1HY)  ILIST = 1
162.          IPLOT = 0
163.          IF (IP .EQ. 1HY)  IPLOT = 1
164.    C
165.          RETURN
166.    C
167.     9000 FORMAT (3F.0)
168.     9005 FORMAT (I,F.0,F.0)
169.     9010 FORMAT (F.0)
170.     9015 FORMAT (F.0)
171.     9020 FORMAT (F.0)
172.     9025 FORMAT (F.0)
173.     9055 FORMAT (I)
174.     9065 FORMAT (A)
175.     9070 FORMAT (I,6F.0)
176.     9075 FORMAT (I)
177.     9080 FORMAT (A1,1X,A1,1X,2F.0)
178.    C
179.     9400 FORMAT (/, ENTER START, END, INCREMENT OF PERIOD RANGE (SEC) 0.)
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180.       9405 FORMAT (/'   ENTER SEA SPECTRUM TYPE AND PARAMETERS'
181.          +           /'        = 1.0          0 '
182.          +           /'        PIERSON-MOSKOWITZ 1, WIND SPEED (KNOTS)'
183.          +           /'        BRETSCHNEIDER     2, SIGNIF WAVE HT (FT), SIGNIF'
184.          +         ' WAVE PERIOD (SEC)'
185.          +           /'        I.S.S.C.          3, SIGNIF WAVE HT (FT), SIGNIF'
186.          +         ' WAVE PERIOD (SEC)'              )
187.       9410 FORMAT (/'   ENTER WATER PLANE RADIUS AT SURFACE (FT) 0')
188.       9415 FORMAT (/'   ENTER DEPTH TO KEEL (FT) 0')
189.       9420 FORMAT (/'   ENTER EXPECTED AVERAGE AMPLITUDE (FT) 0')
190.       9425 FORMAT (/'   ENTER EXPECTED AVERAGE ROLL (DEG) 0')
191.       9455 FORMAT (/'   ENTER NUMBER OF BUOY PARTS 0')
192.       9460 FORMAT (/'   *** FOR EACH PART NUMBER YOU MUST ENTER '/
193.          +         '           SOME IDENTIFIER BEFORE YOU RETURN.'/
194.          +         '           THEN ENTER  K,W,H,T,D,X,F              '/
195.          +         '           K = SHAPE CODE  1 = HOLLOW CYLINDER    '/
196.          +         '                           2 = SOLID CYLINDER     '/
197.          +         '                           3 = DISC               '/
198.          +         '                           4 = TRIANGULAR (RT) PLATE '/
199.          +         '                           5 = RECTANGULAR PLATE'/
200.          +         '           W = WIDTH OR OUTSIDE DIAMETER (FT)     '/
201.          +         '           H = HEIGHT (FT)                        '/
202.          +         '           T = THICKNESS (IN)                     '/
203.          +         '               A -1 ENTERED FOR CASE K=1 WILL ASSUME '/
204.          +         '                 A SOLID (T = W/2/12)             '/
205.          +         '               FOR CASES K=2,3 ENTER ANYTHING     '/
206.          +         '           D = DENSITY (LBM/FT**3)                '/
207.          +         '           X = DISTANCE FROM KEEL TO PART CG (FEET) '/
208.          +         '           F = FOR PLATES ONLY, FRACTIONAL AREA'/
209.          +         '                   OF THE PLATE NORMAL TO MOTION'/
210.          +         '               ENTER 1 FOR CYLINDERS              '////)
211.       9465 FORMAT (/ '   PART NO. ', I3,'0')
212.       9470 FORMAT ( '   ENTER K,W,H,T,D,X,F')
213.       9475 FORMAT (/'   ENTER PART NUMBER TO CHANGE (-1 TO STOP) 0')
214.       9480 FORMAT (/'   ENTER Y OR N FOR LIST AND PLOT OPTIONS'/
215.          +         '           FOR PLOT YOU MAY ALSO ENTER RAD MIN AND MAX 0')
216.    C
217.       9700 FORMAT (/, *** YOURE KIDDING - THE CODES ONLY GO FROM 1 TO 5 '/
218.          +         '           TRY AGAIN '/)
219.       9705 FORMAT (/' *** WHAT KIND OF SHAPE IS THIS 0')
220.       9710 FORMAT (/, *** WHAT DO YOU HAVE IN THERE 0')
221.       9715 FORMAT (/' *** WHERE IS IT 0')
222.       9720 FORMAT (/' *** RANGE OF F = 0.0 TO 1.0 0')
223.       9725 FORMAT (/, *** ONLY ', I3,' COMPONENTS ARE ALLOWED '/
224.          +         '           BUOY DEFINITION TERMINATES '/
225.          +         '           BUOY CHANGES WILL PROCEED  '/)
226.    C
227.          END
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1.          SUBROUTINE BINPUT
2.     C
3.     C              VERSION 1.0        SEP, 1976        R. GOLDSMITH
4.     C
5.     C           THIS ROUTINE INPUTS DATA FOR THE ROLL RAO IN BATCH MODE
6.     C
7.     C
8.     C
9.     C
10.         DIMENSION ICOMMENT(4), ID(10)
11.    C
12.         COMMON / INDEV / NCR,NLP
13.         COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
14.         COMMON / SEASTATE / ISEASEL,WINDV,WAVEHT,WAVEPER,
15.        +             WINDVP4,WAVEHTP2,WAVPERP4
16.         COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
17.        +             DENSITY(50),DISTCGK(50),FRACNORM(50)
18.         COMMON / BOUTS / VOLUME(50),WEIGHT(50)
19.         COMMON / CONSTANT / PI,HT8D,RHO,G
20.         COMMON / BUHY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
21.        +             DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
22.         COMMON / OUTPUTS / ILIST,IPLOT,RMIN,RMAX
23.    C
24.    C
25.         DATA ID /'  H CYL ','  S CYL ','   DISC ',' TRI PLT',' RCT PLT'/
26.    C
27.    C              INPUT TIME AND RANGE
28.         READ (NCR,9005)  T1,T2,T3
29.         IF (T1 .LT. 0.0) GO TO 175
30.         TIME1 = T1
31.         TIME2 = T2
32.         IF (TIME2 .LT. TIME1)  TIME2 = TIME1
33.         IF (T3 .LE. 0.0)  T3 = TIMEDEL
34.         TIMEDEL = T3
35.     175 CONTINUE
36.         WRITE (NLP,9405)  TIME1,TIME2,TIMEDEL
37.    C
38.    C              SELECT SEA STATE PARAMETERS
39.     200 CONTINUE
40.         READ (NCR,9010)  ISTEST, WAVEHT, WAVEPER
41.         IF (ISTEST .GT. 3)  ISTEST = -1
42.         IF (ISTEST .LT. 0)  GO TO 275
43.         ISEASEL = ISTEST
44.         IF (ISEASEL .EQ. 1)  WINDV = WAVEHT
45.         WINDVP4 = WINDV**4
46.         WAVEHTP2 = WAVEHT*WAVEHT
47.         WAVPERP4 = WAVEPER**4
48.     275 CONTINUE
49.         IF (ISEASEL .EQ. 0)   WRITE (NLP,9410)
50.         IF (ISEASEL .EQ. 1)   WRITE (NLP,9411)  WINDV
51.         IF (ISEASEL .EQ. 2)   WRITE (NLP,9412)  WAVEHT,WAVEPER
52.         IF (ISEASEL .EQ. 3)   WRITE (NLP,9413)  WAVEHT,WAVEPER
53.    C
54.    C              ENTER WATER PLANE RADIUS
55.     300 CONTINUE
56.         READ (NCR,9015)  RWLTEST
57.         IF (RWLTEST .LT. 0.0)  GO TO 375
58.         RWL = RWLTEST
59.     375 CONTINUE
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60.           WRITE (NLP,4415)  RWL
61.       C
62.       C               INPUT DEPTH TO KEEL
63.       400 CONTINUE
64.           READ (NCR,9020)  ZKTEST
65.           IF (ZKTEST .LT. 0.0)  GO TO 475
66.           DEPTHK = ZKTEST
67.       475 CONTINUE
68.           WRITE (NLP,9420)  DEPTHK
69.       C
70.       C               ENTER ESTIMATED AVERAGE AMPLITUDE
71.       500 CONTINUE
72.           READ (NCR,9025)  AMPTEM
73.           IF (AMPTEM .LT. 0.0)  GO TO 575
74.           AVERGAMP = AMPTEM
75.       575 CONTINUE
76.           WRITE (NLP,9425)  AVERGAMP
77.       C
78.       C               ENTER ESTIMATED AVERAGE ROLL
79.       600 CONTINUE
80.           READ (NCR,9030)  THETATEM
81.           IF (THETATEM .LT. 0.0)  GO TO 675
82.           THETABAR = THETATEM/RTOD
83.       675 CONTINUE
84.           WRITE (NLP,9430)  THETABAR*RTOD
85.       C
86.       C
87.       700 CONTINUE
88.       C
89.       C               DEFINE BUOY
90.       C
91.           N = 0
92.      1000 CONTINUE
93.           READ (NCR,9055)  NPTEM
94.           IF (NPTEM .LE. 0)  GO TO 2000
95.       C
96.           WRITE (NLP,9455)  NPTEM
97.           WRITE (NLP,9460)
98.           N = 1
99.       C
100.      C               LOOP ON ENTRY
101.     1050 CONTINUE
102.          IF (N .GT. NPMAX)  WRITE (NLP,9725)  NPMAX ,  STOP 1050
103.          READ (NCR,9070)  K,W,H,T,D,X,F,ICOMMENT
104.      C
105.      C               SET INPUTS FOR CORRECT SHAPE
106.          IF ((K .GT. 5) .OR. (K .LT. 1))  OUTPUT K ,  WRITE (NLP,9700) ,
107.       +                                   STOP 1050
108.          IF (W .LE. 0.0)  WRITE (NLP,9705) ,  OUTPUT W ,  STOP 1050
109.          WIDTH(N) = W
110.          IF (H .LT. 0.0)  WRITE (NLP,9705) ,  OUTPUT H ,  STOP 1050
111.          HEIGHT(N) = H
112.          IF (D .LT. 0.0)  WRITE (NLP,9710) ,  OUTPUT D ,  STOP 1050
113.          DENSITY(N) = D
114.          IF (X .LT. 0.0)  WRITE (NLP,9715) ,  OUTPUT X ,  STOP 1050
115.          DISTCGK(N) = X
116.          GO TO (1100,1100,1100,1400,1500), K
117.      C
118.      C               CYLINDERS
119.     1100 CONTINUE
```

```
120.          IF (T .EQ. -1.0)  T = W*6.0
121.          IF ((K .EQ. 2) .OR. (K .EQ. 3) )  T = W*6.0
122.          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  STOP 1100
123.          THICK(N) = T/12.0
124.          FRACNORM(N) = 1.0
125.          ISHAPE(N) = 1
126.          GO TO 1800
127.    C
128.    C                   TRIANGLE
129.     1400 CONTINUE
130.          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  STOP 1400
131.          THICK(N) = T/12.0
132.          IF ((F .GT. 1.0) .OR. (F .LT. 0.0))  WRITE (NLP,9720) ;  OUTPUT F
133.         +                                         ;  STOP 1400
134.          FRACNORM(N) = F
135.          ISHAPE(N) = 2
136.          GO TO 1800
137.    C
138.    C                   RECTANGLE
139.     1500 CONTINUE
140.          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  STOP 1500
141.          THICK(N) = T/12.0
142.          IF ((F .GT. 1.0) .OR. (F .LT. 0.0))  WRITE (NLP,9720) ;  OUTPUT F
143.         +                                         ;  STOP 1500
144.          FRACNORM(N) = F
145.          ISHAPE(N) = 3
146.          GO TO 1800
147.    C
148.    C             COMPUTE VOLUME AND WEIGHT
149.     1800 CONTINUE
150.          CALL BODYVOL (ISHAPE(N),H,W,THICK(N),V)
151.          WEIGHT(N) = V*D
152.          VOLUME(N) = V
153.    C
154.          WRITE (NLP,9465)  N,ID(2*K-1),ID(2*K),W,H,THICK(N),D,X,F
155.         +                  ,ICOMMENT
156.    C             CHECK NUMBER OF ENTRIES
157.          IF (N .GE. NPTEM)  GO TO 2000
158.          N = N + 1
159.          GO TO 1050
160.    C
161.    C             PART CHANGE
162.    C
163.     2000 CONTINUE
164.          IF (NPTEM .GT. 0)  NPARTS = NPTEM
165.          NPTEM = 0
166.          NPARTS = MAX (NPARTS,N)
167.          READ (NCR,9075)  N
168.          IF ((N .GT. 0) .AND. (N .LE. NPARTS + 1))  WRITE (NLP,9470) ;
169.         +                                         GO TO 1050
170.    C
171.     3000 CONTINUE
172.          READ (NCR,9080)  IL,IP,RMIN,RMAX
173.          ILIST = 0
174.          IF (IL .EQ. 1HY)  ILIST = 1
175.          IPLOT = 0
176.          IF (IP .EQ. 1HY)  IPLOT = 1
177.    C
178.          RETURN
179.    C
```

```
180.      9005 FORMAT (3F.0)
181.      9010 FORMAT (I,F.0,F.0)
182.      9015 FORMAT (F.0)
183.      9020 FORMAT (F.0)
184.      9025 FORMAT (F.0)
185.      9030 FORMAT (F.0)
186.      9055 FORMAT (I)
187.      9070 FORMAT (I,6F.0,3A4,A2)
188.      9075 FORMAT (I)
189.      9080 FORMAT (A1,1X,A1,1X,2F.0)
190.   C
191.      9405 FORMAT (/'   PERIOD RANGE, IN SECONDS    START      END      DELTA'/
192.         +          26X,F8.3,F8.3,2X,F8.3)
193.      9410 FORMAT (/'   SEA SPECTRUM = 1.0')
194.      9411 FORMAT (/'   PIERSON-MOSKOWITZ SEA SPECTRUM '
195.         +          /'      WIND SPEED = ',F10.3,' KNOTS')
196.      9412 FORMAT (/'   BRETSCHNEIDER SEA SPECTRUM'
197.         +          /'      SIGNIFICANT WAVE HT = ',F10.3,' FEET'
198.         +          /'      SIGNIFICANT WAVE PERIOD = ',F10.3,' SEC')
199.      9413 FORMAT (/'   I.S.S.C. SEA SPECTRUM'
200.         +          /'      SIGNIFICANT WAVE HT = ',F10.3,' FEET'
201.         +          /'      SIGNIFICANT WAVE PERIOD = ',F10.3,' SEC.)
202.      9415 FORMAT (/'   WATER PLANE RADIUS AT SURFACE = ',F6.2,' FT')
203.      9420 FORMAT (/'   DEPTH TO THE KEEL = ',F6.2,' FT')
204.      9425 FORMAT (/'   ESTIMATED AVERAGE AMPLITUDE = ,F6.2,' FT')
205.      9430 FORMAT (/'   ESTIMATED AVERAGE ROLL = ',F6.2,' DEG')
206.      9455 FORMAT (/'   NUMBER OF PARTS = ',I3)
207.      9460 FORMAT (//'                                                                '
208.         +          '     C.G. '/
209.         +          '                                                       DENSITY',
210.         +          '     ABOVE'/
211.         +          '   PART                      WIDTH      HEIGHT      THICK      (LBSM/ ',
212.         +          '   KEEL   FRACT'/
213.         +          '   NO      SHAPE      (FT)      (FT)        (FT)       FT**3)',
214.         +          '   (FT)    NORM   COMMENTS'/
215.         +          '   ----  ---------  -------  -------  --------  --------',
216.         +          '   -----  -----  ----------------'/)
217.      9465 FORMAT (I5, 2X,2A4, F8.2, F8.2, F11.4, F10.1, F7.2, F7.2, 2X,4A4)
218.      9470 FORMAT (/)
219.   C
220.      9700 FORMAT (/' *** YOURE KIDDING - THE CODES ONLY GO FROM 1 TO 5 '/)
221.      9705 FORMAT (/, *** WHAT KIND OF SHAPE IS THIS 0 ')
222.      9710 FORMAT (/' *** WHAT DO YOU HAVE IN THERE 0 ')
223.      9715 FORMAT (/' *** WHERE IS IT 0 ')
224.      9720 FORMAT (/, *** RANGE OF F = 0.0 TO 1.0 0 ')
225.      9725 FORMAT (/' *** ONLY ',I3,' COMPONENTS ARE ALLOWED '/
226.         +          '         THE PROGRAM TERMINATES '/)
227.   C
228.         END
```

```
1.              SUBROUTINE BODYMI (IS,H,W,T,PMI)
2.      C
3.      C                   VERSION 1.0         SEP, 1976          R. GOLDSMITH
4.      C
5.      C             THIS RHUTINE COMPUTES THE BASIC SHAPE MOMENT OF INERTIA
6.      C                ASSUMING UNIFORM DENSITY
7.      C
8.              COMMON / IBDEV / NCR,NLP
9.      C
10.     C
11.             IF ((IS .GT. 3) .OR. (IS .LT. 1))  WRITE (NLP,9700)   IS J
12.            +                                                      STOP 10
13.     C
14.             GO TO (100,200,300), IS
15.     C
16.     C                   CYLINDER
17.       100 CONTINUE
18.             WT = W - 2.0*T
19.             IF (WT .LT. 0.0)  WT = 0.0
20.             PMI = (W*W + WT*WT)/16.0 + H*H/12.0
21.             RETURN
22.     C
23.     C                   TRIANGLES
24.       200 CONTINUE
25.     C ***MOST PLATES ARE SMALL SO THE INERTIA IS IGNORED FOR NOW
26.             PMI = 0.0
27.             RETURN
28.     C
29.     C                   RECTANGULAR PLATE
30.       300 CONTINUE
31.             PMI = H*H/12.0
32.             RETURN
33.     C
34.     C
35.      9700 FORMAT (///'   *** WHAT KIND OF SHAPE IS CODE ', I6/
36.            +          '            PROGRAM STOPS IN RHUTINE BODYMI')
37.     C
38.             END
```

```fortran
1.            SUBROUTINE BODYVOL (IS,H,W,T,V)
2.       C
3.       C            VERSION 1.0        SEP, 1976          R. GOLDSMITH
4.       C
5.       C            COMPUTE THE BODY VOLUME
6.       C
7.            COMMON / IODEV / NCR,NLP
8.            COMMON / CONSTANT / PI,RTOD,RHO,G
9.       C
10.      C
11.           IF ((IS .GT. 3) .OR. (IS .LT. 1))  WRITE (NLP,9700)  IS ,
12.          +                                                      STOP 10
13.           GO TO (100,200,300), IS
14.      C
15.      C
16.      C                CYLINDERS
17.       100 CONTINUE
18.           WT = W - 2.0*T
19.           IF (WT .LT. 0.0)  WT = 0.0
20.           V = H*PI*(W*W - WT*WT)/4.0
21.           RETURN
22.      C
23.      C            TRIANGLES
24.       200 CONTINUE
25.           V = 0.5*H*W*T
26.           RETURN
27.      C
28.      C            RECTANGULAR PLATE
29.       300 CONTINUE
30.           V = H*W*T
31.           RETURN
32.      C
33.      C
34.      9700 FORMAT (//'    *** WHAT IN THE WORLD IS SHAPE CODE ', I6/
35.          +        '               PROGRAM STOPS IN ROUTINE BODYVOL')
36.      C
37.           END
```

```
1.                SUBROUTINE DISPLACE
2.       C
3.       C                    VERSION 1.0        SEP, 1976         R. GOLDSMITH
4.       C
5.       C            THIS ROUTINE IS USED TO COMPUTE THOSE PARAMETERS
6.       C               ASSOCIATED WITH THE BUOY DISPLACEMENT
7.       C
8.            COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
9.          +                 DENSITY(50),DISTCGK(50),FRACNORM(50)
10.           COMMON / ROUTS / VOLUME(50),WEIGHT(50)
11.           COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
12.          +                 DEPTHB(50),DEPTHT(50)
13.           COMMON / CONSTANT / PI,RTHO,RHO,G
14.           COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
15.          +                 DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
16.           COMMON / MOMENTS / BUOYMI,ADDMI,VIRTINRT,WATERIM,BUOYMR,
17.          +                 BUOYMOT,BUOYMD,WATERMD,DAMPM
18.       C
19.       C
20.           RHOG = RHO*G
21.           WDISPLAC = 0.0
22.           SUMCBK = 0.0
23.           ADDMI = 0.0
24.       C
25.       C            BEGIN LOOP ON EACH PART
26.       C
27.           DO 1000  I = 1,NPARTS
28.             IS = ISHAPE(I)
29.             H = HEIGHT(I)
30.             W = WIDTH(I)
31.             T = THICK(I)
32.             V = VOLUME(I)
33.             X = DISTCGK(I)
34.             F = WEIGHT(I)
35.             WD(I) = W
36.             HD(I) = H
37.             XD(I) = X
38.       C
39.       C            COMPUTE DEPTH OF PART BOTTOM AND TOP
40.       C
41.           DCG = DEPTHK - X
42.       C                  CHECK SHAPE
43.           GO TO (100,200,100), IS
44.       C                          CYLINDER AND RECTANGULAR PLATE
45.     100   CONTINUE
46.           TEM = H/2.0
47.           DB = DCG + TEM
48.           DT = DCG - TEM
49.           GO TO 300
50.       C                          TRIANGLE
51.     200   CONTINUE
52.           DB = DCG + H/3.0
53.           DT = DCG - H*0.6667
54.       C    ***       THE ABOVE IS ONLY A GUESS - CORRECT
55.           GO TO 300
56.       C
57.       C            IF PART TOTALLY OUT OF WATER IGNORE
58.     300   CONTINUE
59.           IF (DB .LE. 0.0)  VCOR = V ,  GO TO 750
```

```
60.    C              IF PART TOP IS IN WATER OK
61.                   IF (DT .GE. 0.0)  VCOR = 0.0 ,  GO TO 750
62.    C
63.    C              COMPUTE VOLUME CORRECTION FOR OUT OF WATER
64.                   IF ((IS .NE. 1) .AND. (IS .NE. 4))  GO TO 350
65.    C              CYLINDER AND RECTANGULAR PLATE MOD
66.                   XD(I) = DEPTHK - DB/2.0
67.                   HD(I) = DB
68.                   GO TO 600
69.    C              TRIANGLE MOD
70.        350   CONTINUE
71.                   IF (IS .NE. 2)  GO TO 400
72.    C ****   XD(I) SHOULD ALSO CHANGE IF MI IS TO BE CORRECT ALSO HD
73.                   WD(I) = H*W/ABS(DT)
74.                   GO TO 600
75.        400   CONTINUE
76.    C
77.        600   CONTINUE
78.                   CALL BODYVOL (IS,ABS(DT),WD(I),T,VCOR)
79.                   DT = 0.0
80.    C
81.    C              VOLUME IN WATER
82.        750   CONTINUE
83.                   VD(I) = V - VCOR
84.                   DEPTHB(I) = DB
85.                   DEPTHT(I) = DT
86.    C
87.    C              WEIGHT OF WATER
88.                   WGTW = VD(I)*RHOG
89.                   WDISPLAC = WDISPLAC + WGTW
90.                   SUMCBK = SUMCBK + WGTW*XD(I)
91.                   FD(I) = WGTW
92.    C
93.    C              COMPUTE THE ADDED MI OF THE WATER BODY
94.    C              ABOUT ITS OWN AXIS
95.                   CALL BODYMI (IS,HD(I),WD(I),T,WBODYMI)
96.                   ADWMI = VD(I)*RHO* (XD(I) - BUOYCGK)**2
97.                   ADDMI = ADDMI + WGTW*WBODYMI/G + ADWMI
98.    C
99.     1000 CONTINUE
100.   C
101.   C              COMPUTE THE CENTER OF BUOYANCY
102.   C
103.               BUOYCBK = SUMCBK/WDISPLAC
104.               DEPTHCB = DEPTHK - BUOYCBK
105.   C
106.               RETURN
107.   C
108.               END
```

```
1.        SUBROUTINE BUOYDAMP
2.     C
3.     C          VERSION 1.0        SEP, 1976        R. GOLDSMITH
4.     C
5.     C          THIS ROUTINE COMPUTES THE BUOY DAMPING MOMENT
6.     C             ASSUMPTIONS 1- THAT THE ANGLE OF ROLL IS 'SMALL'
7.     C                            SO THAT THE HORIZONTAL COMPONENT OF
8.     C                            BUOY MOTION AND WATER VELOCITY HAVE
9.     C                            PERPENDICULAR EFFECT.
10.    C                          - DAMPING FORCE IS LINEAR AND PROPORTION
11.    C                            . TO SPEED
12.    C
13.    C
14.          COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
15.          COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
16.         +               DENSITY(50),DISTCGK(50),FRACNORM(50)
17.          COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
18.         +               DEPTHB(50),DEPTHT(50)
19.          COMMON / CONSTANT / PI,RTOD,RHO,G
20.          COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
21.         +               DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
22.          COMMON / CEFS / DRAG(5),COEFM(5)
23.          COMMON / MOMENTS / BUOYMI,AUDMI,VIRTINRT,WATERIM,BUOYMR,
24.         +               BUOYMDT,BUOYMD,WATERMD,DAMPM
25.    C
26.    C
27.          ALPHAT= 4.0*RHO*THETABAR/(3.0*PI)
28.    C
29.    C          THIS SECTION COMPUTES JUST THE BUOY DAMPING
30.    C
31.          BUOYMDT = 0.0
32.          DO 50 I = 1,NPARTS
33.    C          CHECK IF ITS OUT OF WATER
34.          IF (VD(I) .LE. 0.0)  GO TO 50
35.    C          CHECK SHAPE
36.          GO TO  (20,30,30), ISHAPE(I)
37.    C                    CYLINDER
38.    20    CONTINUE
39.          XB = DEPTHB(I) - DEPTHCG
40.          XT = DEPTHT(I) - DEPTHCG
41.          PMU = 0.25*WIDTH(I)*(SIGN (XB**4,XB) - SIGN (XT**4,XT) )
42.          GO TO 40
43.    C                    TRIANGULAR RECTANGULAR PLATES
44.    30    CONTINUE
45.          PAREA = VD(I)/THICK(I)
46.          XC = BUOYCGK - XD(I)
47.          PMU = PAREA*XC*XC*FRACNORM(I)
48.          GO TO 40
49.    40    CONTINUE
50.          ALPHA = ALPHAT*DRAG(ISHAPE(I))
51.          BUOYMDT = BUOYMDT + ALPHA*PMD
52.    X     OUTPUT ALPHA*PMD
53.    50 CONTINUE
54.    C          **********************************************
55.    C          NOTE: TO SIMPLIFY THE COMPUTATION THIS TERM HAS BEEN
56.    C                COMPUTED AS A CONSTANT.  THE FREQUENCY
57.    C                CONTRIBUTION IS MULTIPLIED IN AT THE BEGINNING
58.    C                OF THE MAIN FREQUENCY ITERATION IN THE MAIN
59.    C                PROGRAM.
```

```
60.   C            ************************************************
61.            RETURN
62.   C
63.   C
      C            **********************************************
64.   C
65.            ENTRY WATERDAMP
66.   C
67.   C                    WATER MOMENT OF DAMPING
68.   C
69.            BETAT = 4.0*RHO*AVERGAMP*FREQ/(3.0*PI)
70.            WATERMD = 0.0
71.   C
72.   C                    COMPUTE DAMPING
73.            DO 1000  I = 1,NPARTS
74.   C                       CHECK IF ITS OUT OF WATER
75.            IF (VD(I) .LE. 0.0)  GO TO 1000
76.   C                    CHECK SHAPE
77.            GO TO (100,300,300), ISHAPE(I)
78.   C                    CYLINDERS
79.      100   CONTINUE
80.            XB = DEPTHB(I) - DEPTHCG
81.            XT = DEPTHT(I) - DEPTHCG
82.            WAVEXB = WAVEN*XB*2.0
83.            WAVEXT = WAVEN*XT*2.0
84.            WAVENP2 = WAVEN*WAVEN
85.            TERMB = ((-WAVEXB - 1.0)*EXP (-WAVEXB) + 1.0)
86.            TERMT = ((-WAVEXT - 1.0)*EXP (-WAVEXT) + 1.0)
87.            WMD =  (TERMB - TERMT)*WIDTH(I)*EXP (-2.0*WAVEN*DEPTHCG)
88.         +            /(4.0*WAVENP2)
89.            GO TO 900
90.   C
91.   C                    TRIANGULAR AND RECTANGULAR PLATES
92.      300   CONTINUE
93.            PAREA = VD(I)/THICK(I)
94.            XC = BUOYCGK - XD(I)
95.            ZC = DEPTHK - XD(I)
96.   C                    WATER MOMENT
97.            WMD = PAREA*XC*FRACNORM(I)*EXP (-WAVEN*ZC)
98.            GO TO 900
99.      900   CONTINUE
100.  C
101.  C            GET TOTAL DAMPING
102.           BETA = BETAT*DRAG(ISHAPE(I))
103.           WMD = BETA*WMD
104.           WATERMD = WATERMD + WMD
105.  1000 CONTINUE
106.  C
107.  C            TOTAL DAMPING MOMENT
108.  C
109.           DAMPM = BUOYMD + WATERMD
110.  C
111.           RETURN
112.  C
113.           END
```

```
1.          SUBROUTINE WATRINRT
2.     C
3.     C              VERSION 1.0         SEP, 1976        R. GOLDSMITH
4.     C
5.     C          THIS ROUTINE COMPUTES THE COEFFICIENT FOR THE INERTIA
6.     C              MOMENT DUE TO WATER PARTICLE ACCELERATION.
7.     C
8.     C
9.          COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
10.         COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
11.        +              DENSITY(50),DISTCGK(50),FRACNORM(50)
12.         COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
13.        +              DEPTHB(50),DEPTHT(50)
14.         COMMON / CONSTANT / PI,RTOD,RHO,G
15.         COMMON / BUHY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
16.        +              DEPTHK,BUBYCGK,DEPTHCG,BUBYCBK,DEPTHCB,WDISPLAC
17.         COMMON / CREFS    DRAG(5),COEFM(5)
18.         COMMON / MOMENTS / BUBYMI,ADDMI,VIRTINRT,WATERIM,BUBYMR,
19.        +              BUBYMOT,BUBYMD,WATERMD,DAMPM
20.    C
21.         BETAT = PI*RHO/4.0
22.         CONSI = BETAT/(WAVEN*WAVEN)*EXP (-WAVEN*DEPTHCG)
23.    C
24.         WATERIM = 0.0
25.    C
26.    C              COMPUTE WATER PARTICLE INERTIA MOMENT
27.         DO 1000  I = 1,NPARTS
28.    C              CHECK IF ITS OUT OF WATER
29.         IF (VD(I) .LE. 0.0)  GO TO 1000
30.    C              CHECK SHAPE
31.         GO TO (100,300,300),  ISHAPE(I)
32.    C
33.    C              CYLINDERS
34.    100    CONTINUE
35.         XB = DEPTHB(I) - DEPTHCG
36.         XT = DEPTHT(I) - DEPTHCG
37.         WAVENXB = WAVEN*XB
38.         WAVENXT = WAVEN*XT
39.         TERM1B = (-WAVENXB - 1.0)*EXP (-WAVENXB)
40.         TERM1B = + TERM1B + 1.0
41.         TERM1T = (-WAVENXT - 1.0)*EXP (-WAVENXT)
42.         TERM1T = + TERM1T + 1.0
43.         WMI = CONST*(WIDTH(I)*WIDTH(I) )*(TERM1B - TERM1T)
44.         GO TO 900
45.    C
46.    C              TRIANGULAR AND RECTANGULAR PLATES
47.    300    CONTINUE
48.    C ****CURRENTLY SET TO ZERO
49.         WMI = 0.0
50.         GO TO 900
51.    C
52.    900    CONTINUE
53.         WMI = -WMI*COEFM(ISHAPE(I))
54.         WATERIM = WATERIM + WMI
55.    C
56.    1000 CONTINUE
57.    C
58.         RETURN
59.    C
60.         END
```

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HEAVE AND ROLL RESPONSE OF FREE FLOATING BODIES
OF CYLINDRICAL SHAPE by H. O. Berteaux, R. A. Goldsmith
and W. E. Schott, III. 112 pages. February 1977. Prepared
for the Office of Naval Research under Contract N00014-75-C-1064
NR 294-044 and from the NOAA Data Buoy Office.

The following report describes a computer solution
to help predict the heave and roll response of free floating
bodies of cylindrical shape when excited by random seas with
known spectra.

The basic concepts of harmonic analysis and statistics
used in the method are first briefly reviewed. The report
then presents a detailed derivation of the linear heave and
roll response amplitude operators, that is the expressions
of the vertical and angular displacements produced by a
simple harmonic wave of one foot amplitude.

The second part of the report reviews the computation
procedure and the program's logic. It gives a detailed set
of instructions for the program users, reviews the program's
capabilities and limitations, and presents three case studies.

The heave and roll response programs are written for
use with XEROX SIGMA 7 computers. Program listings are
given in the appendix.

1. Spar buoys
2. Buoy Dynamics
3. Heave and Roll of Spar Buoys
I. Berteaux, H. O.
II. Goldsmith, R. A.
III. Schott, W. E., III
IV. N00014-75-C-1064 NR 294-044
V. NOAA Data Buoy Office

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